

**N94- 25114**

# **Optimal RTLS Abort Trajectories for an HL-20 Personnel Launch Vehicle**

**Kevin Dutton  
Spacecraft Controls Branch**

## **Outline**

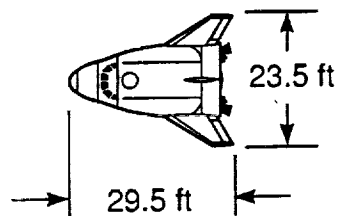
- **Objective of study**
- **HL-20 Vehicle and Mission**
- **Modelling Information**
- **Problem Formulation**
- **Solution Method**
- **Results**
- **Concluding Remarks**

# Objective

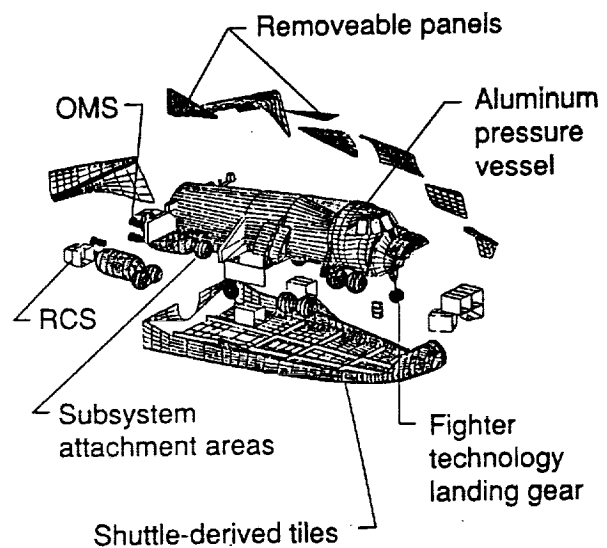
**Primary: Determine whether RTLS abort at T seconds along launch trajectory is possible using optimal control theory**

**Secondary: Assess effects of bank angle constraint, lift coefficient constraint, free and fixed final boundary conditions, etc.**

## HL-20 PLS BASELINE DESIGN



	Weight, lb
Dry (with 22% margin)	19,777
Landed	22,057
On-Orbit	26,186
Launch Escape System/Adapter	8,420
Gross Launch on NLS	34,607



## HL-20 PLS CURRENT TECHNOLOGY DESIGN

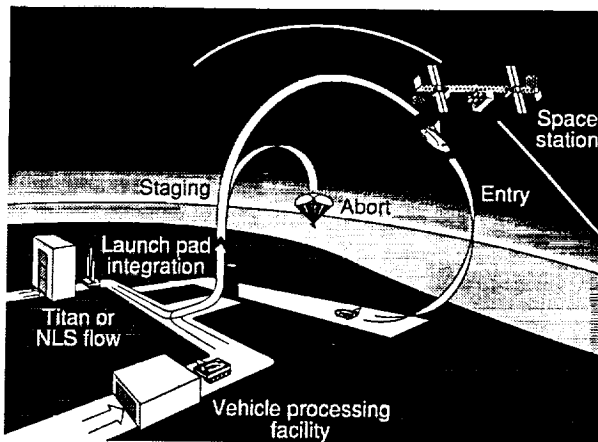
## THE PERSONNEL LAUNCH SYSTEM (PLS)

### Complementary System to Space Shuttle

- Space Station crew transfer
- Alternate access to/from space for people/priority cargo

### Space Station Reference Mission

- Transfer and return up to 8 Space Station personnel and/or priority cargo
- 72-hour mission duration
- 1,100 ft/sec on-orbit propulsive capability
- Placed in orbit by existing or future booster system
- Kennedy Space Center launch/landing site
- Alternate landing site capability



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## HL-20 Aborts

### VAB Analysis

- On the pad
- 0-20 sec      Return to Launchsite (Shuttle landing facility)
- 20-65 sec      RTLS (Skid strip)
- 65-403 sec      Ocean landing by parachute
- 403-478 sec      Transatlantic abort landing
- 478+ sec      Abort to orbit

## Vehicle Aerodynamic Model

- Aerodynamic data from Jackson and Cruz
- At each angle of attack and Mach number, find  $\delta_E, \delta_L, \delta_U$  that trim vehicle and minimize drag; calculate  $C_l$  and  $C_d$  here
- For each Mach number, determine coefficients for  $C_d$  expression
 
$$C_D = C_{D_0}(M) + C_{D_1}(M) C_L + C_{D_2}(M) C_L^2$$

## Optimal Control Theory

- Cost  $\min_{\bar{u}} J = \Phi[\bar{x}(t_0), \bar{x}(t_f)]$
- Plant  $\dot{\bar{x}} = f(\bar{x}, \bar{u})$
- Constraints:
 

Control	State
$\bar{g}(\bar{x}, \bar{u}) = 0$	$\bar{c}(\bar{x}) = 0$
$\bar{h}(\bar{x}, \bar{u}) \leq 0$	$\bar{d}(\bar{x}) \leq 0$
- Boundary conditions  $\psi[\bar{x}(t_0), \bar{x}(t_f)] = 0$

## Cost Function and Plant

- $J = -h(t_f)$  (final altitude)
- States  $\bar{x} = [h \ x \ y \ V \ \gamma \ \psi]^T$ 
  - $x, y$  Cartesian system,  $x$  east,  $y$  north, origin at point runway centerline extended
  - $\psi$  0 for easterly flight, increases CCW
- Controls  $\bar{u} = [C_L \ \sigma]^T$ 
  - $\sigma$  negative for right bank
- Equations of motion: flat earth, non-thrusting, aerospace vehicle

## Control/State Constraints

- Bank angle can be constrained (40 deg. nominal)

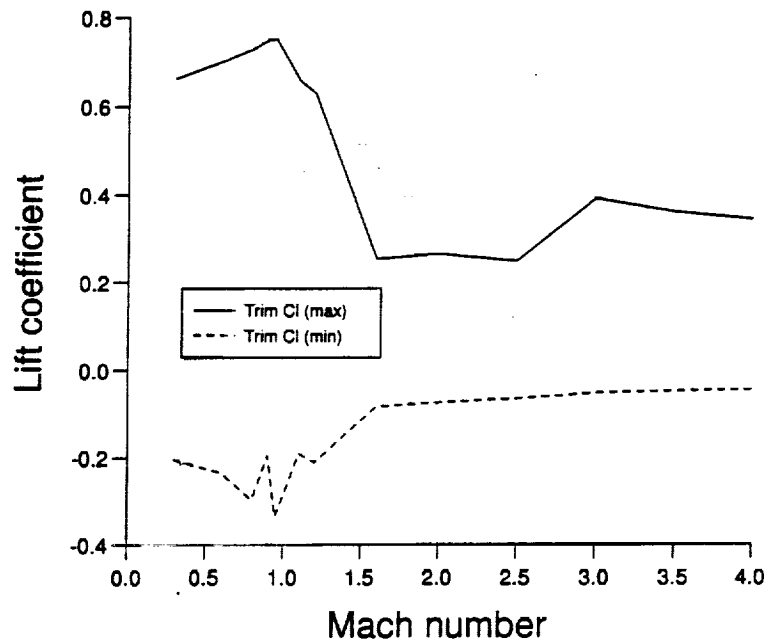
$$-\sigma_{\max} \leq \sigma \leq \sigma_{\max}$$

- Lift coefficient is constrained between upper and lower trim limits (function of Mach)

$$C_{L_{\min}}(M) \leq C_L \leq C_{L_{\max}}(M)$$

- Normal and axial load factor constraints (3 g units nominal)

## Lift Coefficient Constraint

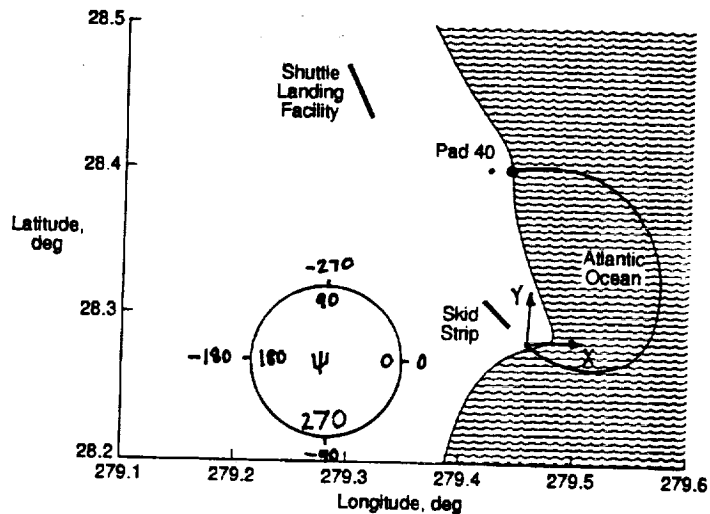


## Initial Conditions

- Initial conditions for abort at T seconds are conditions at T along ascent trajectory followed by primary solid rocket motor (srm) burn, followed by sustainer srm burn
- Example: Initial conditions for abort at T=30
  - $h(t_0) = 32882 \text{ ft}$        $V(t_0) = 1565 \text{ ft/sec}$
  - $x(t_0) = -7409 \text{ ft}$        $\gamma(t_0) = 79.7 \text{ deg}$
  - $y(t_0) = 45357 \text{ ft}$        $\psi(t_0) = -2.0 \text{ deg}$

## Final Conditions

For all cases:  
 $x(t_f) = 0.0 \text{ ft}$   
 $y(t_f) = 0.0 \text{ ft}$   
 $V(t_f) = 521.0 \text{ ft/sec}$   
 $\gamma(t_f) = -19.0 \text{ deg}$   
 $\psi(t_f) = -220.7 \text{ deg}$



## Solution Method

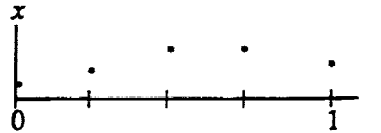
### Trajectory Optimization by Differential Inclusion (TODI)

- eliminates controls from problem by constraining state rates
- leads to nonlinear programming problem where parameters are state values at user defined nodes (NPSOL)

### The Differential Inclusion Approach Explained on a Simple Example

$$\begin{aligned} \min & -x(1) \\ \dot{x} &= u, \quad 0 \leq u \leq 1 \\ x(0) &= 0 \end{aligned}$$

pick states at equidistantly chosen node points



neighboring states have to satisfy either

(differential equation approach)

$$\frac{x_{i+1} - x_i}{\Delta t} = u_i, \quad 0 \leq u_i \leq 1$$

or

(differential inclusion approach)

$$\frac{x_{i+1} - x_i}{\Delta t} \geq 0 \quad \text{and} \quad \frac{x_{i+1} - x_i}{\Delta t} \leq 1$$

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### General Discretization Scheme

Optimal control problem

$$\min_{u \in (PWC[0,1])^m} \Phi(x(0), x(1))$$

$$\Psi(x(0), x(1)) = 0$$

$$\dot{x} = f(x(t), u(t))$$

$$g(x(t), u(t)) = 0$$

$$h(x(t), u(t)) \leq 0$$

$$c(x(t)) = 0$$

$$d(x(t)) \leq 0$$

Finite dimensional discretization

$$\min_{[x_0 \dots x_N] \in \mathbb{R}^{n \times N}} \Phi(x_0, x_N)$$

$$\Psi(x_0, x_N) = 0$$

for  $i = 0, \dots, N-1$ :

$$p\left(\frac{x_{i+1} - x_i}{N}, x_i\right) = 0$$

$$q\left(\frac{x_{i+1} - x_i}{N}, x_i\right) \leq 0$$

for  $i = 0, \dots, N$ :

$$c(x_i) = 0$$

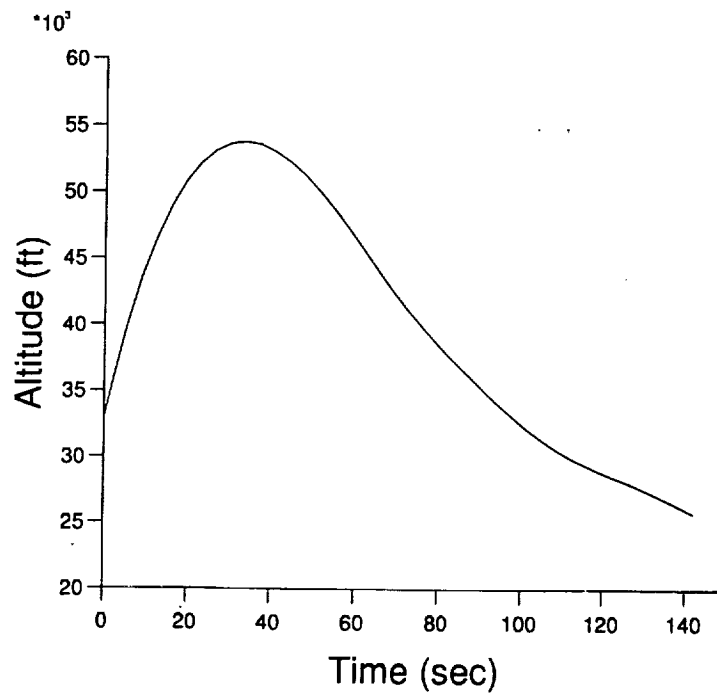
$$d(x_i) \leq 0$$

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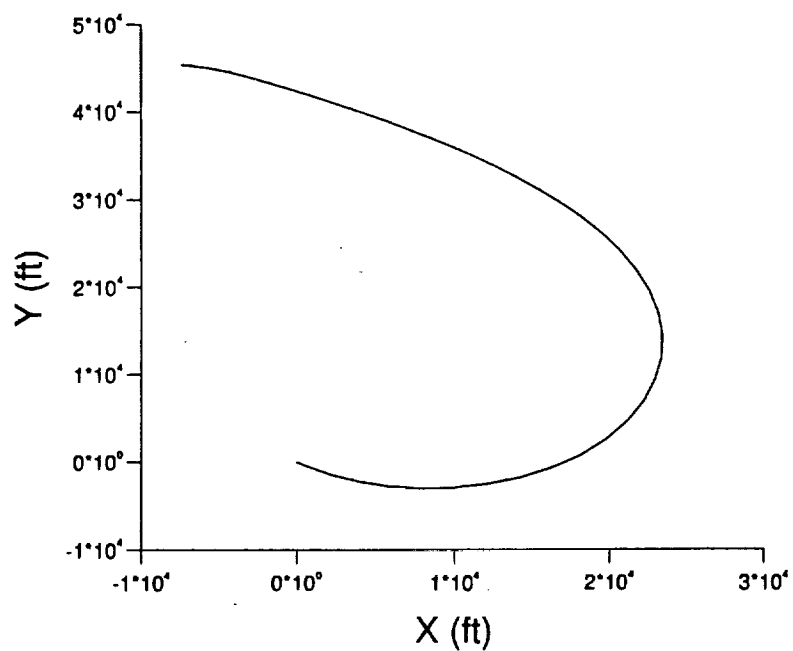
## Solution for 30 Second Abort Case

### Altitude vs. Time



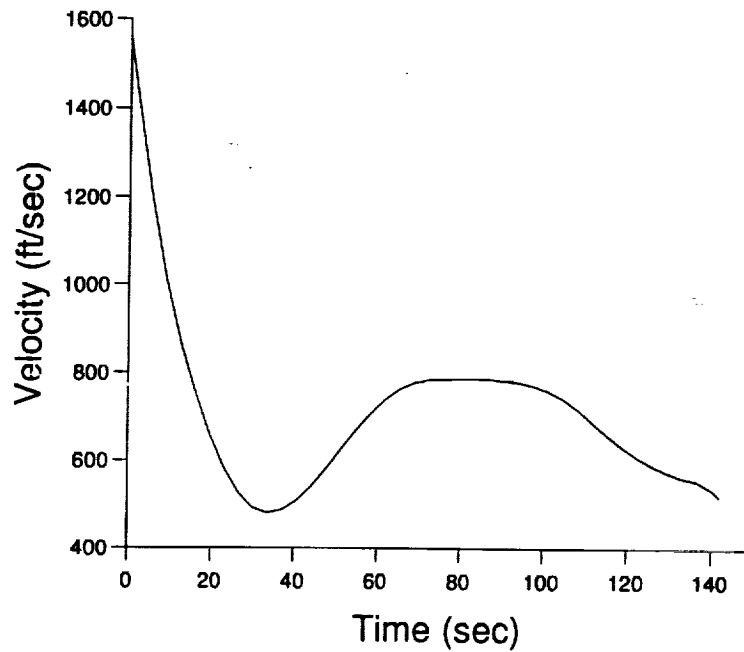
## Solution for 30 Second Abort Case

### Groundtrack



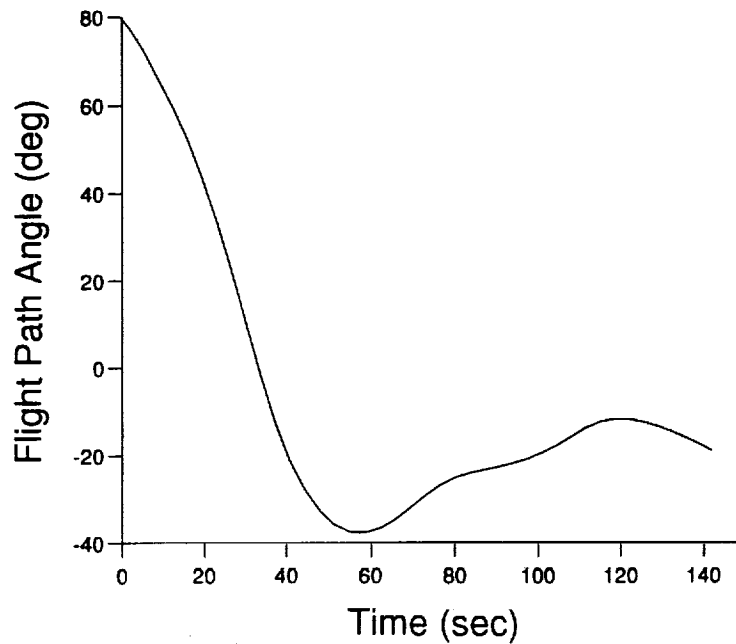
## Solution for 30 Second Abort Case

### Velocity vs. Time



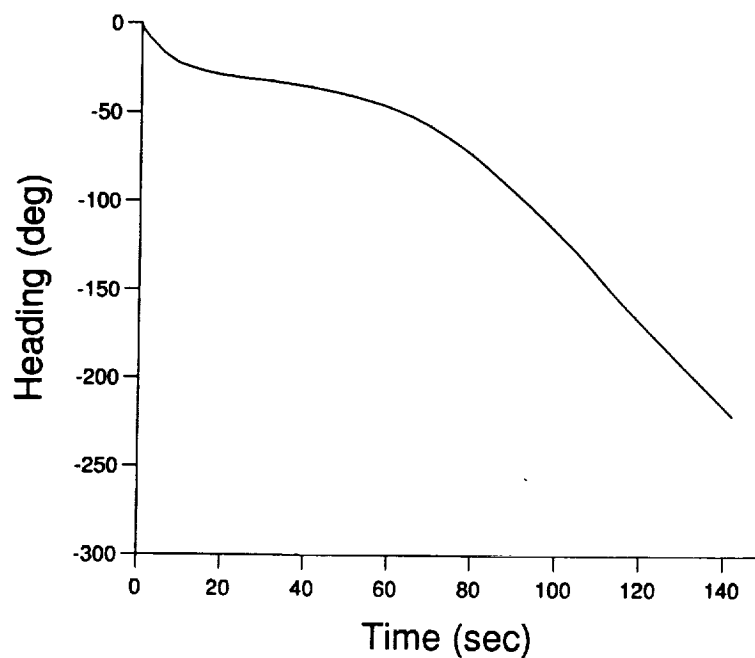
## Solution for 30 Second Abort Case

### Flight Path Angle vs. Time



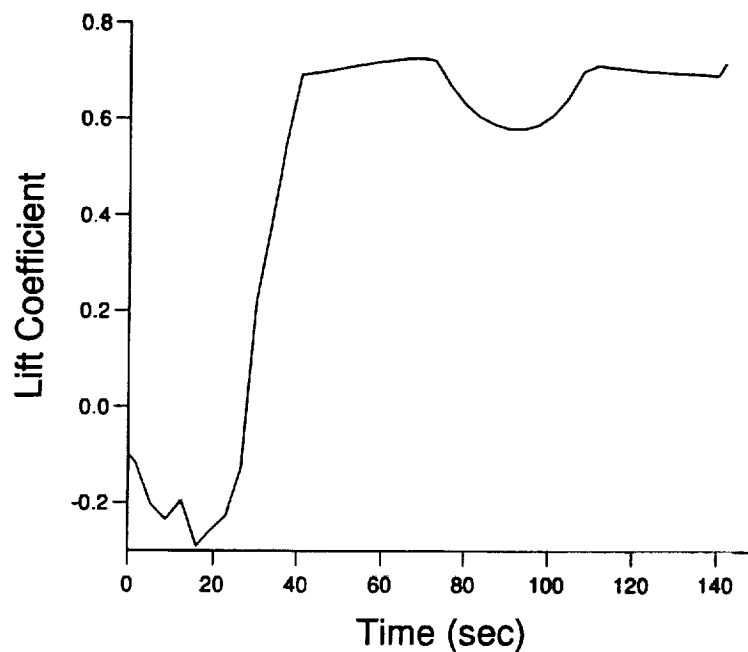
## Solution for 30 Second Abort Case

### Heading vs. Time



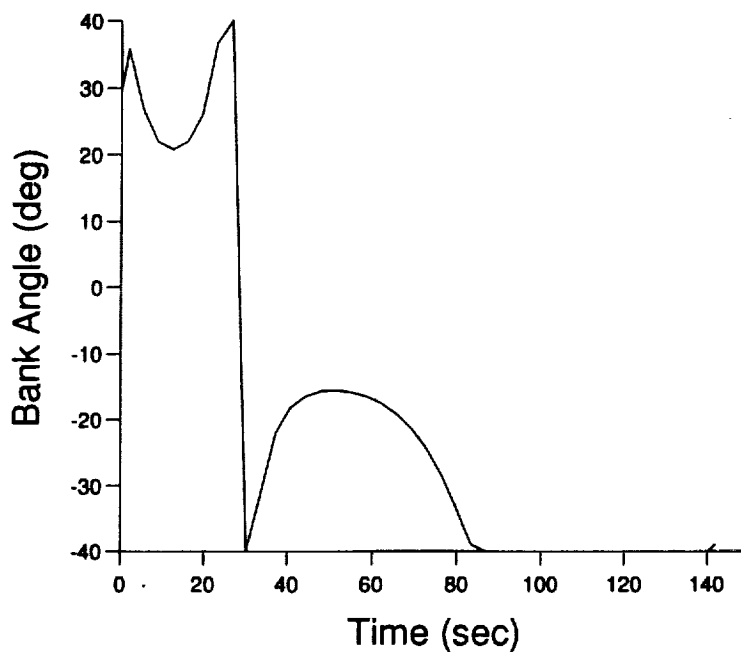
## Solution for 30 Second Abort Case

### Lift Coefficient vs. Time



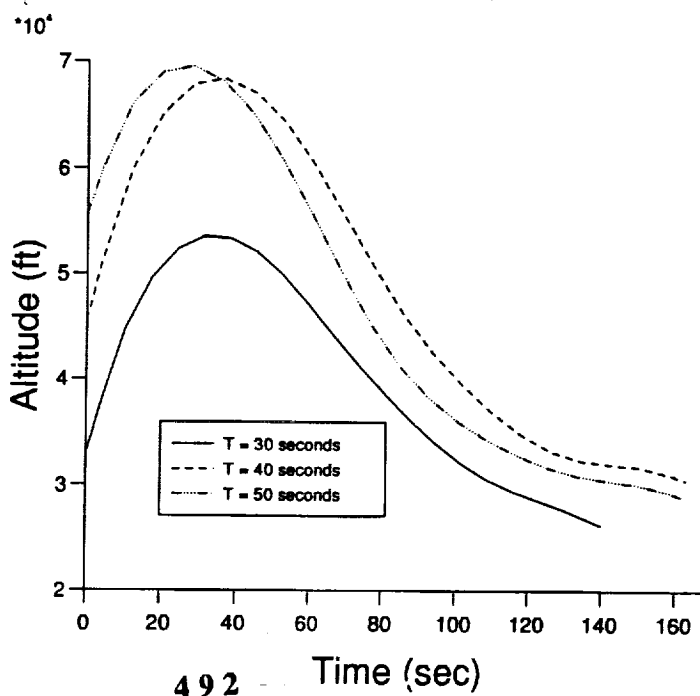
## Solution for 30 Second Case

Bank Angle vs. Time



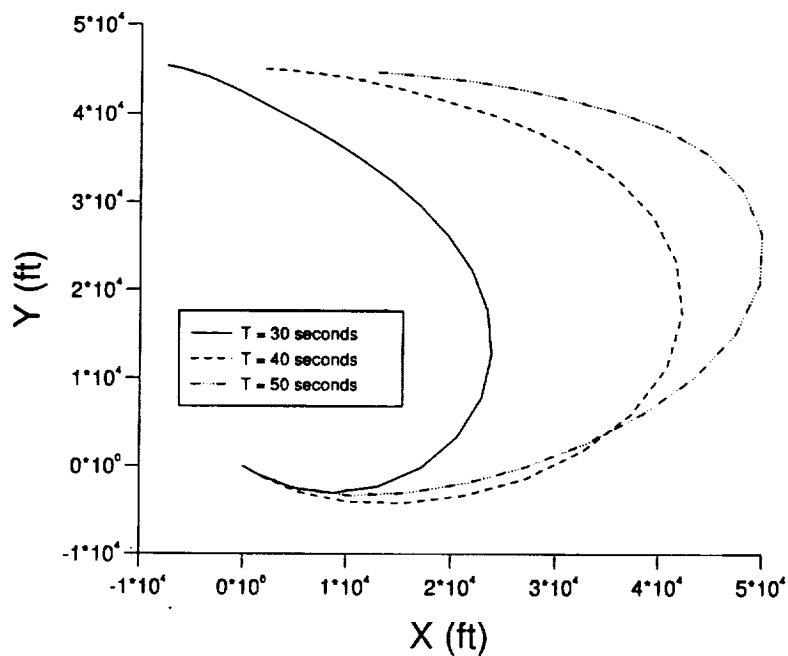
## Comparison of T=30,40,50 Sec. Aborts

Altitude vs. Time



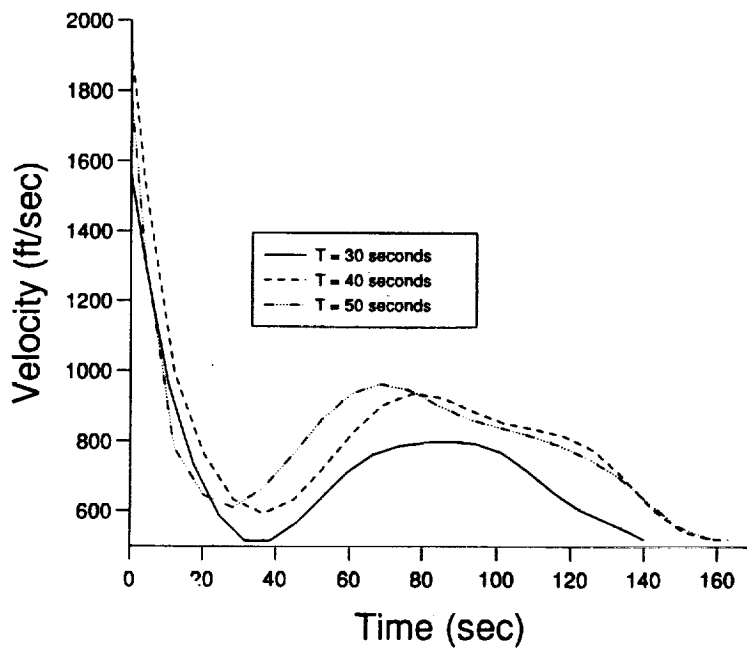
## Comparison of T=30,40,50 Sec. Aborts

### Groundtrack



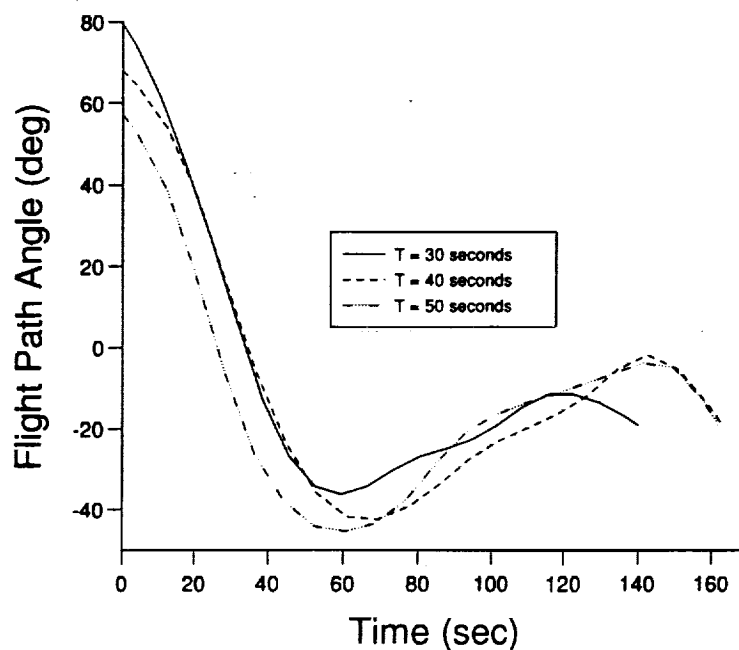
## Comparison of T=30,40,50 Sec. Aborts

### Velocity vs. Time



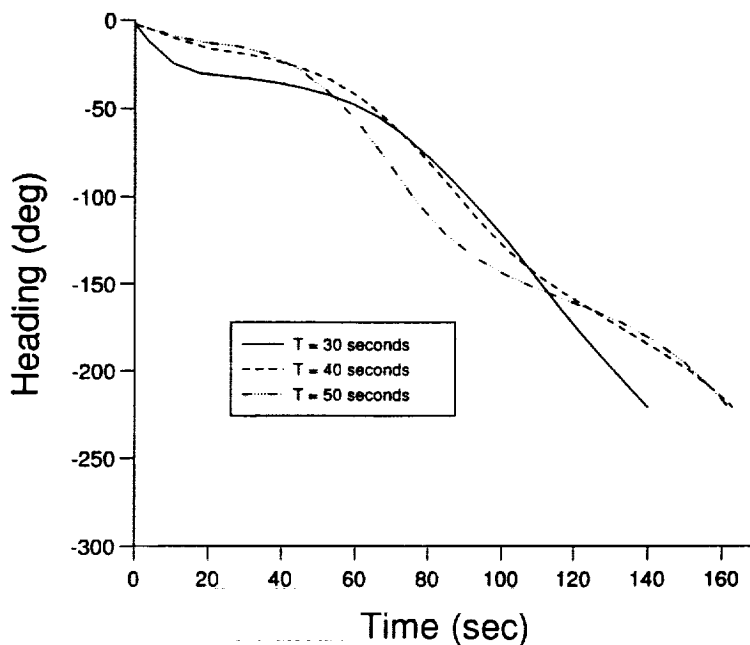
## Comparison of T=30,40,50 Sec. Aborts

### Flight Path Angle vs. Time



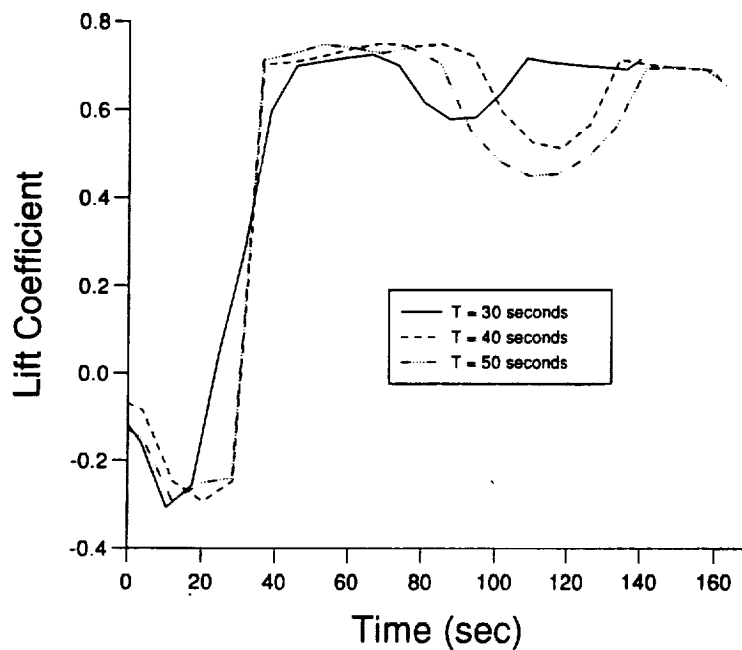
## Comparison of T=30,40,50 Sec. Aborts

### Heading vs. Time



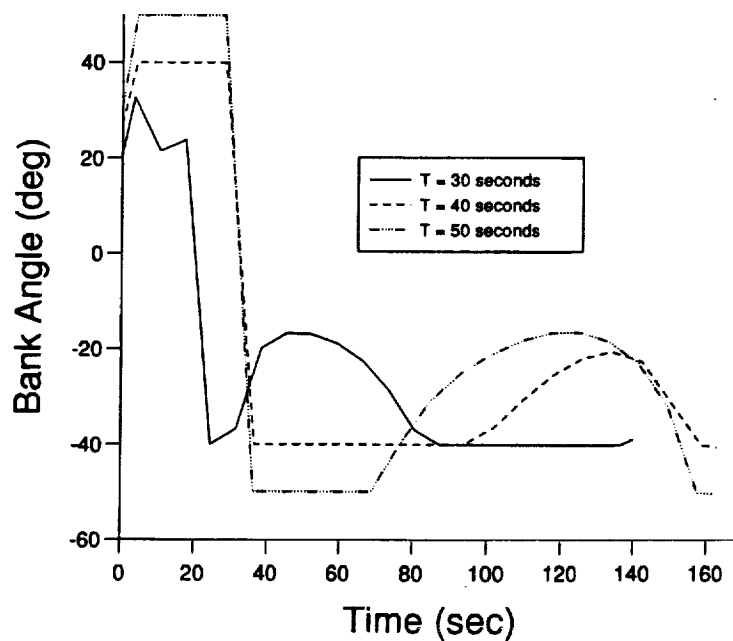
## Comparison of T=30,40,50 Sec. Aborts

### Lift Coefficient vs. Time



## Comparison of T=30,40,50 Sec. Aborts

### Bank Angle vs. Time



## **Concluding Remarks**

- When final  $V$  is fixed, maximizing final  $h$  is nearly same as maximizing final energy  
==> calculation of minimum energy trajectories
- Choice of cost function for abort (and reentry) not obvious
- Future work:
  - Single Stage Vehicle (?)
  - experiment to assess "power" of TODI approach compared to traditional shooting



# **Range Optimal Atmospheric Flight Vehicle Trajectories In Presence of a Dynamic Pressure Limit**

BY

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## **PURPOSE OF THIS TALK**

- Explore nature of range-optimal flight
- Present techniques for identifying temporal structure of optimal control
- Demonstrate in application to an aircraft example

## PROBLEM FORMULATION

### Cost Function:

$$J[u] = -x(t_f)$$

### Initial Conditions:

$$\begin{aligned} E(0) &= 38,029.[m] \\ h(0) &= 12,119.[m] \\ \gamma(0) &= 0.^\circ \\ x(0) &= 0.[m] \end{aligned}$$

### Control constraint:

$$\begin{aligned} 0 &\leq \eta \leq 1 \\ |n| &\leq n_{\max} \end{aligned}$$

### State Equations:

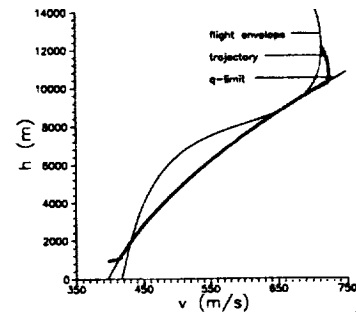
$$\begin{aligned} \dot{E} &= (\eta T - D) \frac{v}{W} \\ \dot{h} &= v \sin \gamma \\ \dot{\gamma} &= \frac{g}{v} (n - \cos \gamma) \\ \dot{x} &= v \cos \gamma \end{aligned}$$

### Final Conditions:

$$\begin{aligned} E(t_f) &= 9000.[m] \\ h(t_f) &= 942.[m] \\ \gamma(t_f) &= -11.5^\circ \\ x(t_f) &\text{ be maximized} \end{aligned}$$

### State constraint:

$$v \leq v_{\max}(h)$$



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## SIGNIFICANCE FOR PRACTICAL APPLICATION

- Validate optimality of solutions obtained with other methods
- Use optimal solutions to develop guidance laws based on neighboring optimal control
- Decide on choice of discretization ( e.g. finite elements)

## OUTLINE

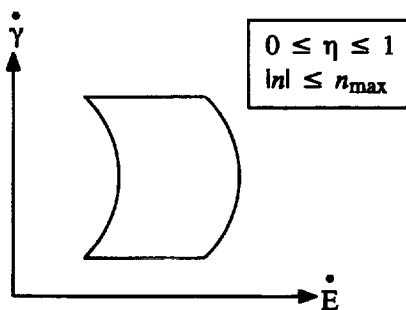
- Hodograph analysis
- Possible control logics / Optimal switching structures
- Numerical procedures and results
- Summary and Conclusions

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## HODOGRAPH ANALYSIS

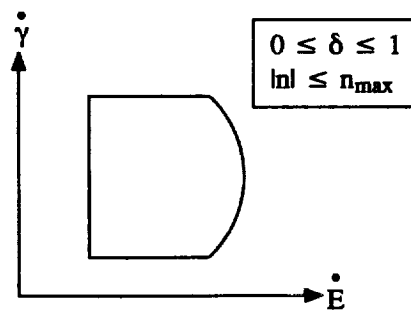
Original formulation:

$$\begin{aligned}\dot{E} &= (\eta T - D) \frac{v}{W} \\ \dot{\gamma} &= \frac{g}{v} (n - \cos \gamma)\end{aligned}$$



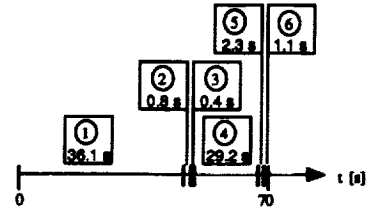
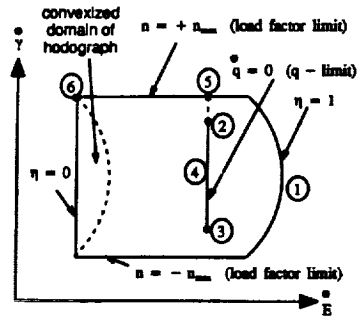
New formulation:

$$\begin{aligned}\dot{E} &= [\delta(T - D + D_{\max}) - D_{\max}] \frac{v}{W} \\ \dot{\gamma} &= \frac{g}{v} (n - \cos \gamma)\end{aligned}$$



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## POSSIBLE CONTROL LOGICS / OPTIMAL SWITCHING STRUCTURE



- |   |   |
|---|---|
| 1) $v - v_{\max} < 0, \delta = 1,$  | $\frac{\partial H}{\partial n} = 0$           |
| 2) $v - v_{\max} = 0, \delta = 1,$  | $n > 0$ from $\frac{d}{dt}(v - v_{\max}) = 0$ |
| 3) $v - v_{\max} = 0, \delta = 1,$  | $n < 0$ from $\frac{d}{dt}(v - v_{\max}) = 0$ |
| 4) $v - v_{\max} = 0, \delta$ from $\frac{d}{dt}(v - v_{\max}) = 0, n$ singular   |   |
| 5) $v - v_{\max} = 0, \delta$ from $\frac{d}{dt}(v - v_{\max}) = 0, n = n_{\max}$ |   |
| 6) $v - v_{\max} < 0,$  | $\delta = 0, n = n_{\max}$                    |

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## THOROUGH ANALYSIS YIELDS

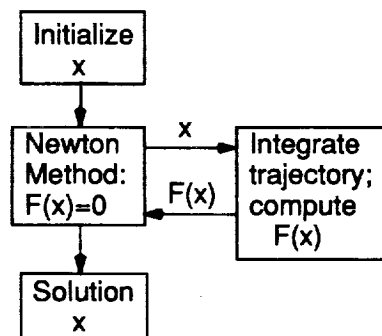
- 12 different possible control logics are obtained
  - 6 cases with  $v_{\max}$ -limit not active
    - 1 first-order singular case with  $v_{\max}$ -limit not active
  - 6 cases with active  $v_{\max}$ -limit
    - 1 first-order singular case with  $v_{\max}$ -limit active
    - 1 second order singular case with  $v_{\max}$ -limit active
- To perform higher order optimality tests the Generalized Legendre-Clebsch condition has been extended to the case of singular control in presence of state/control constraints

### REMARKS

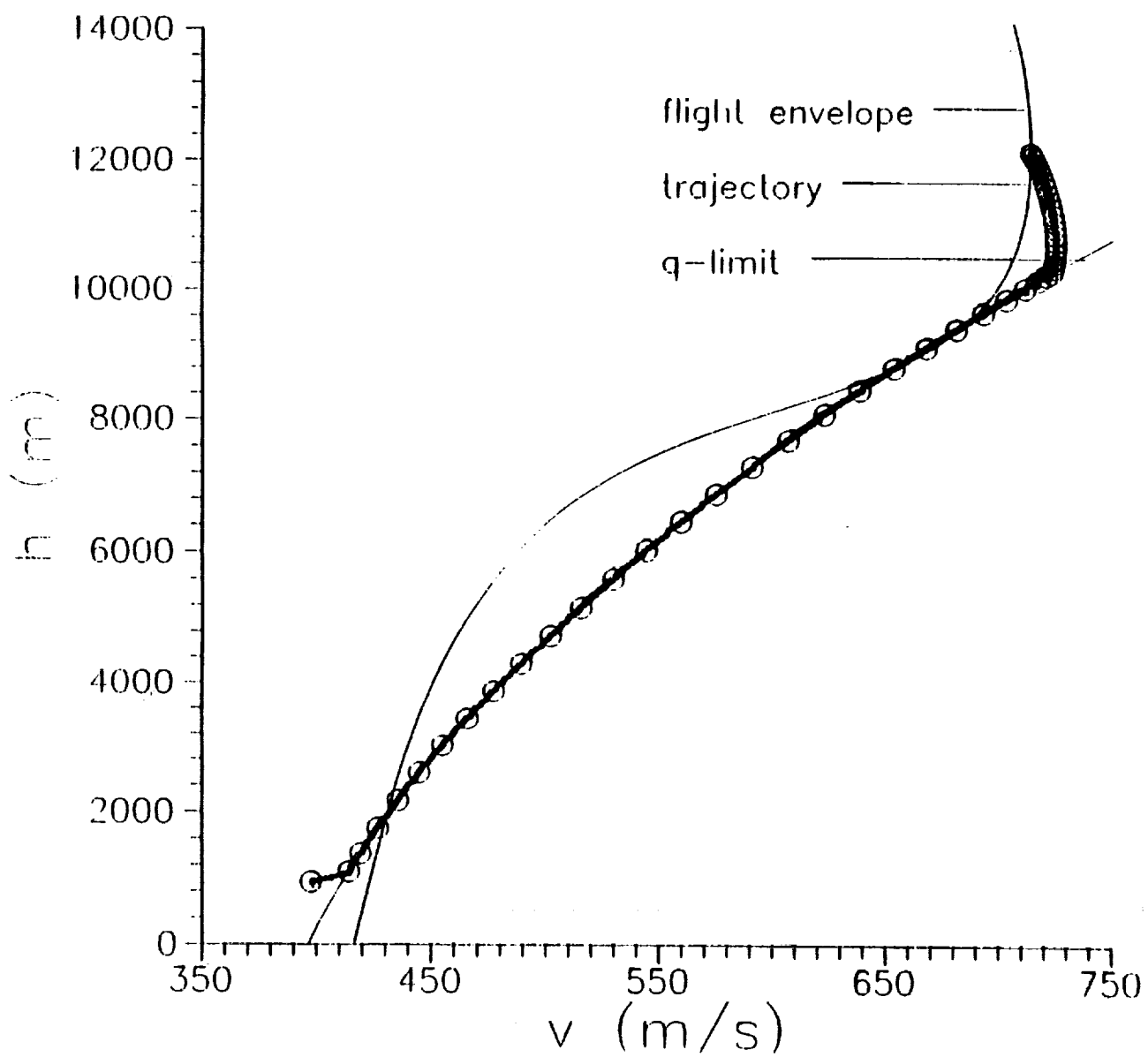
- Switching structure is non-intuitive
- Dynamic pressure constraint makes problem very ill-conditioned
  - (i) standard shooting codes fail
  - (ii) developed flexible shooting code
  - (iii) trick: start integration at the end of singular control

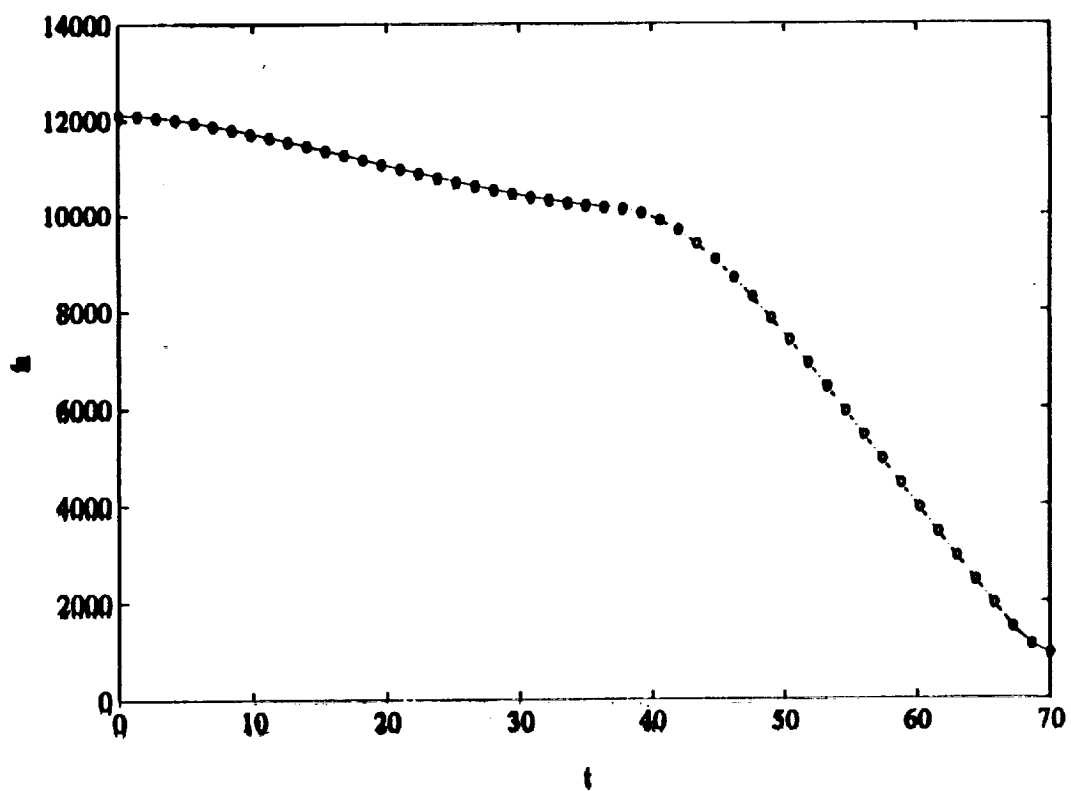
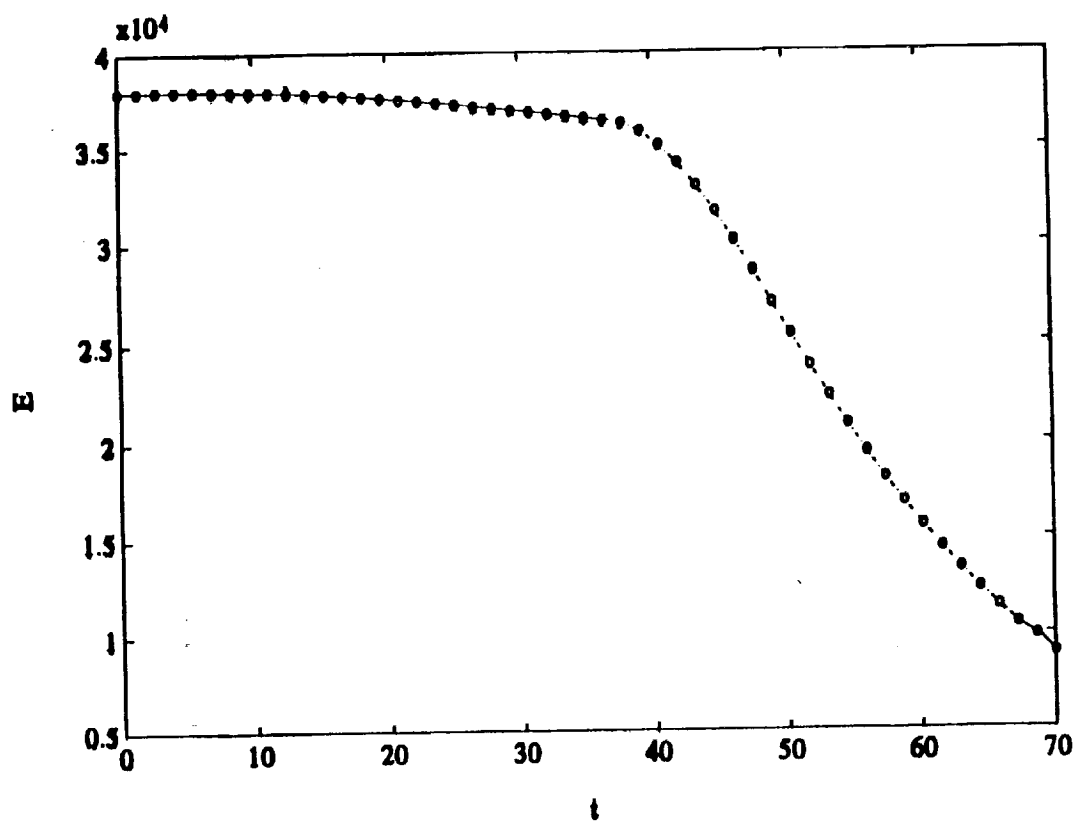
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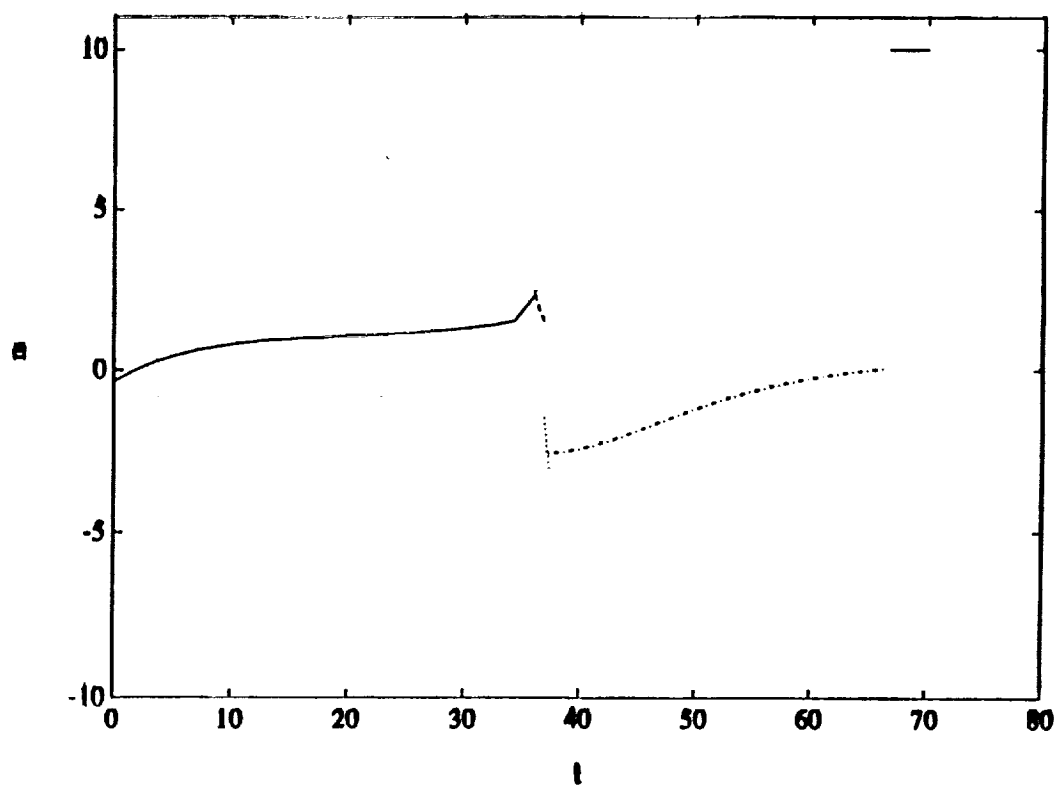
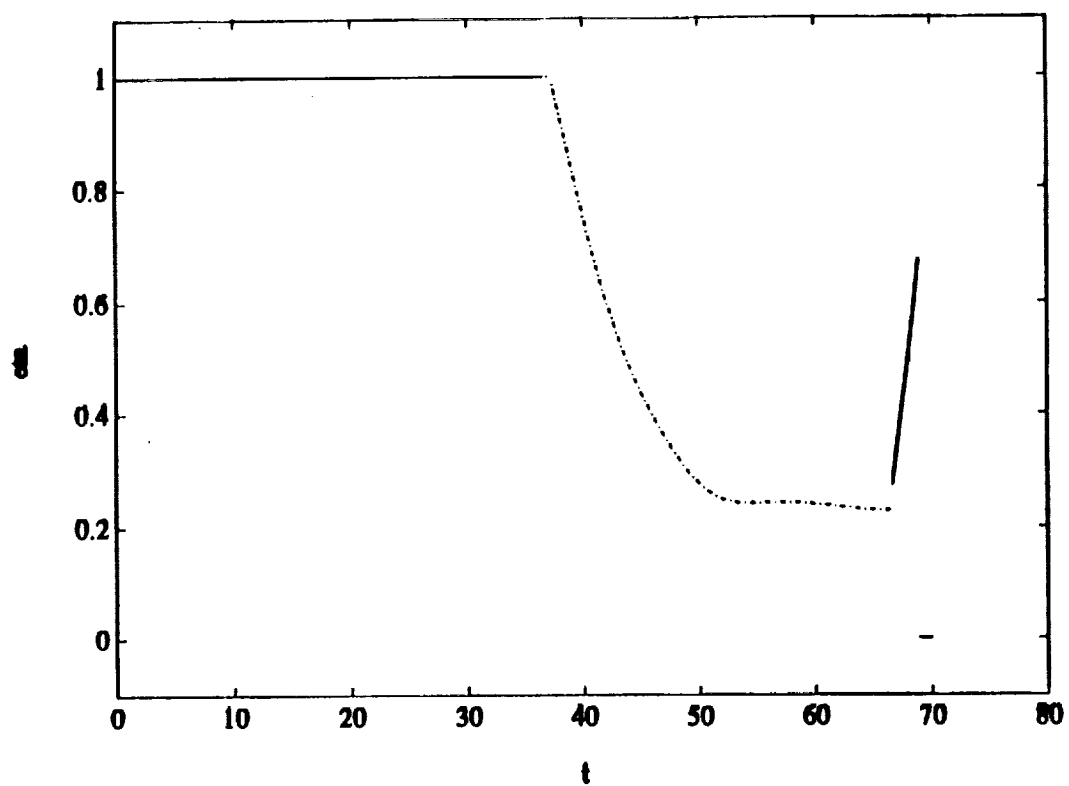
### STRUCTURE OF SHOOTING CODE



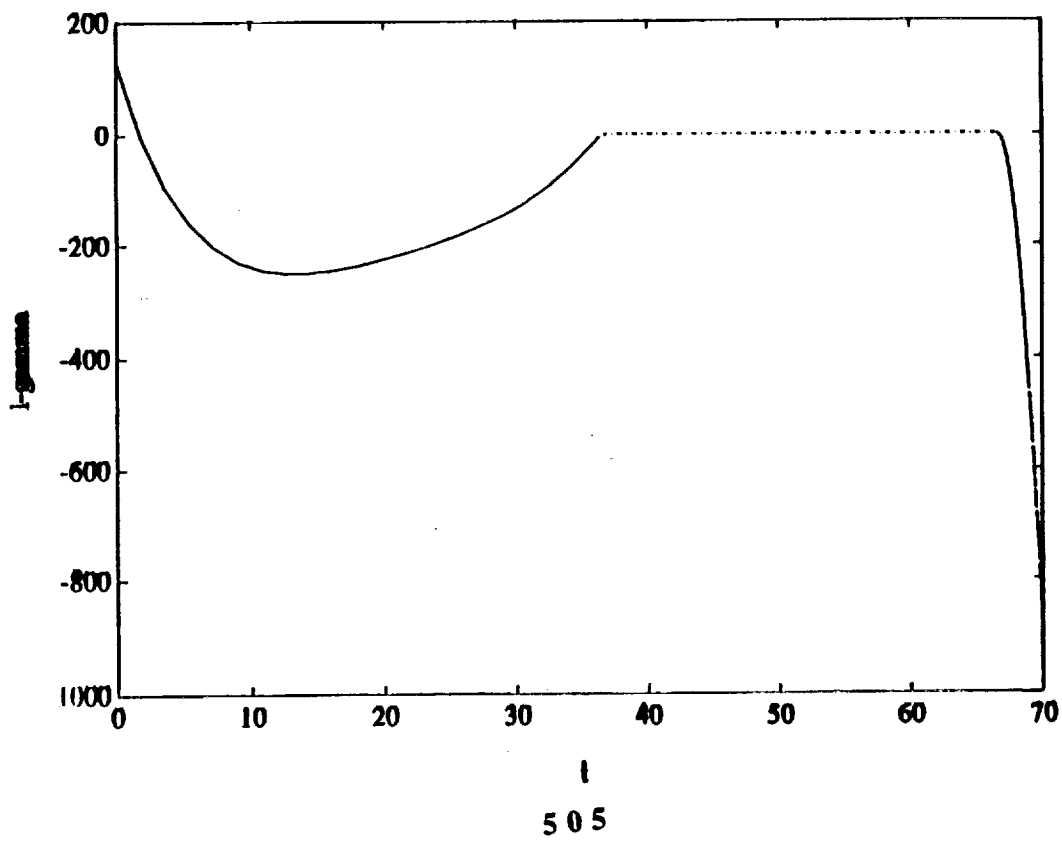
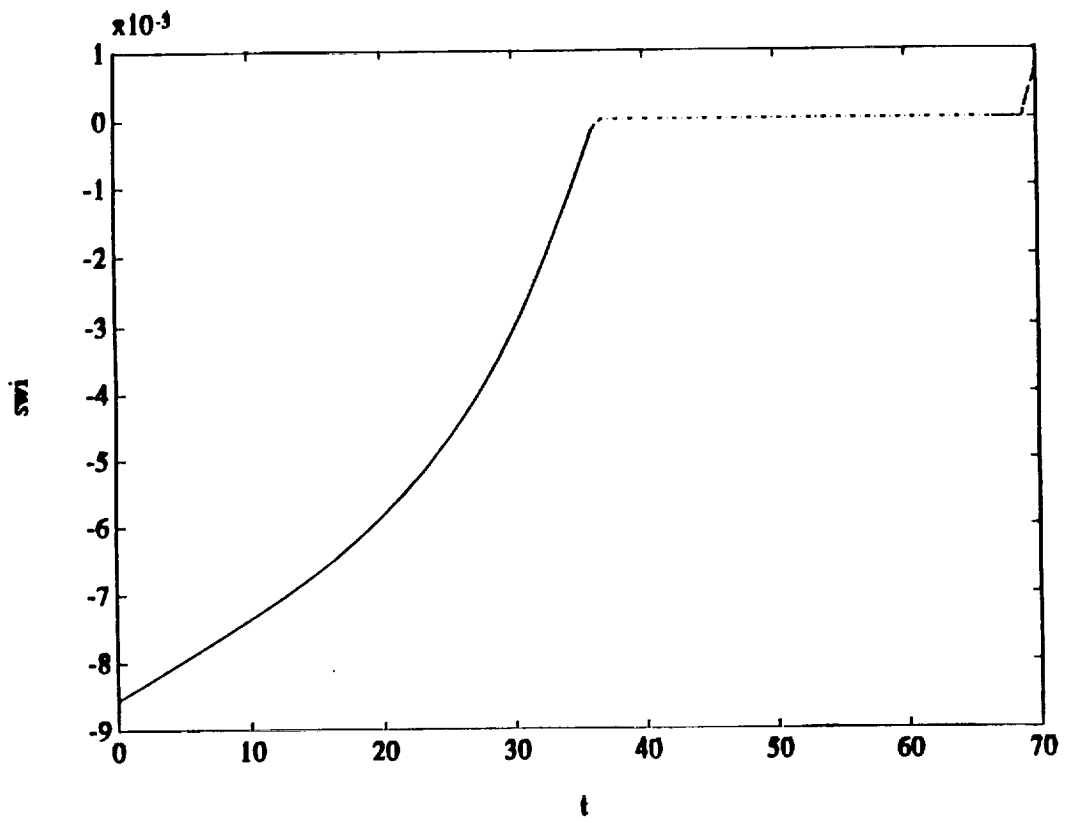
- Boundary value problem: find  $x$  such that  $F(x)=0$
- User completely determines function  $F$
- Simple structure allows independent debugging of  $F(x)$

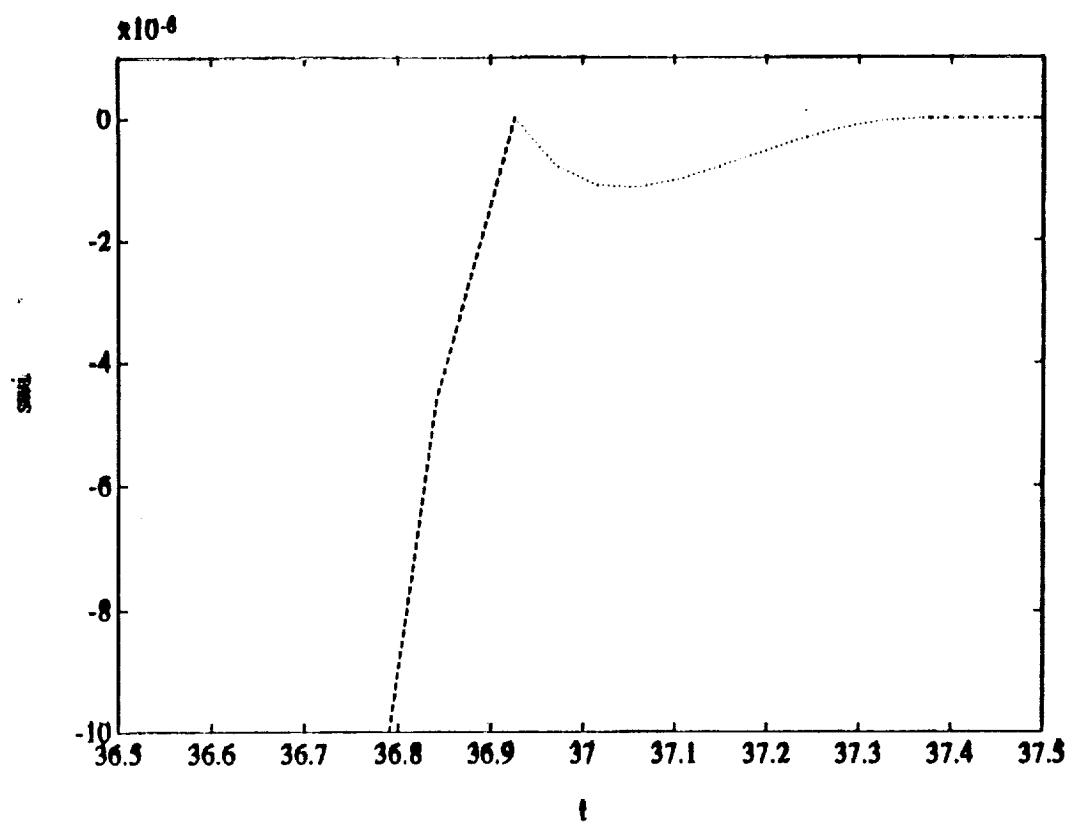
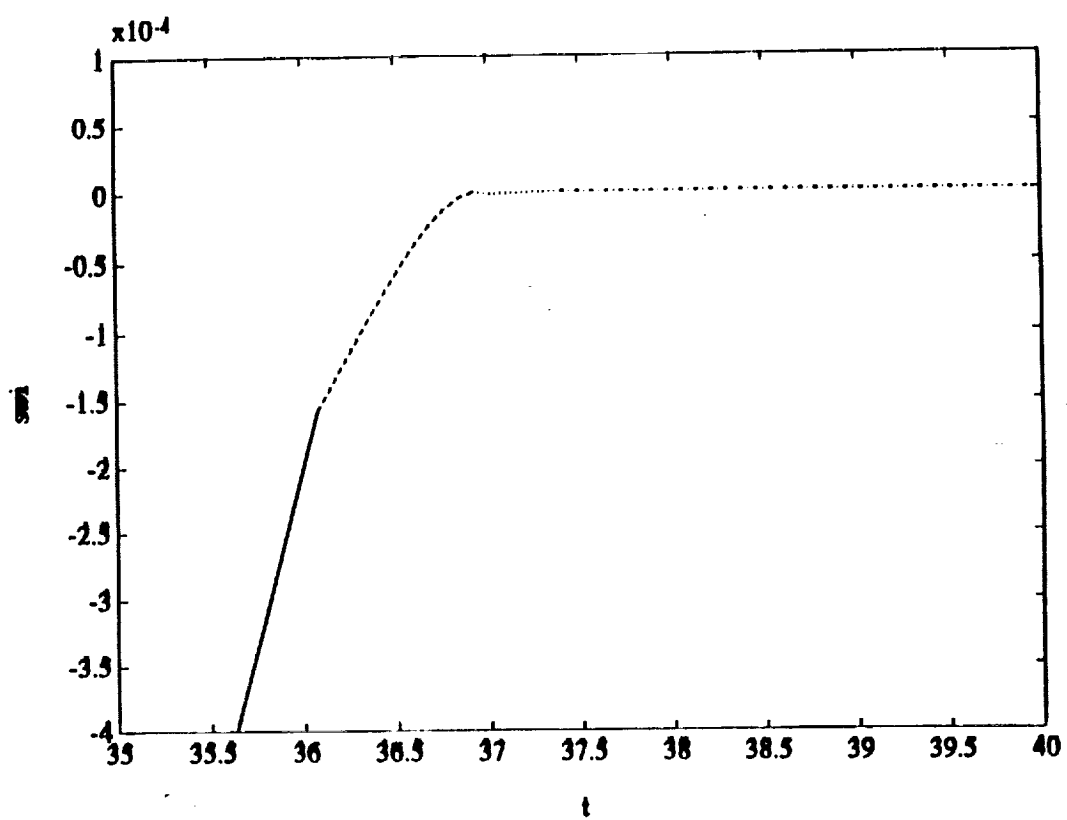


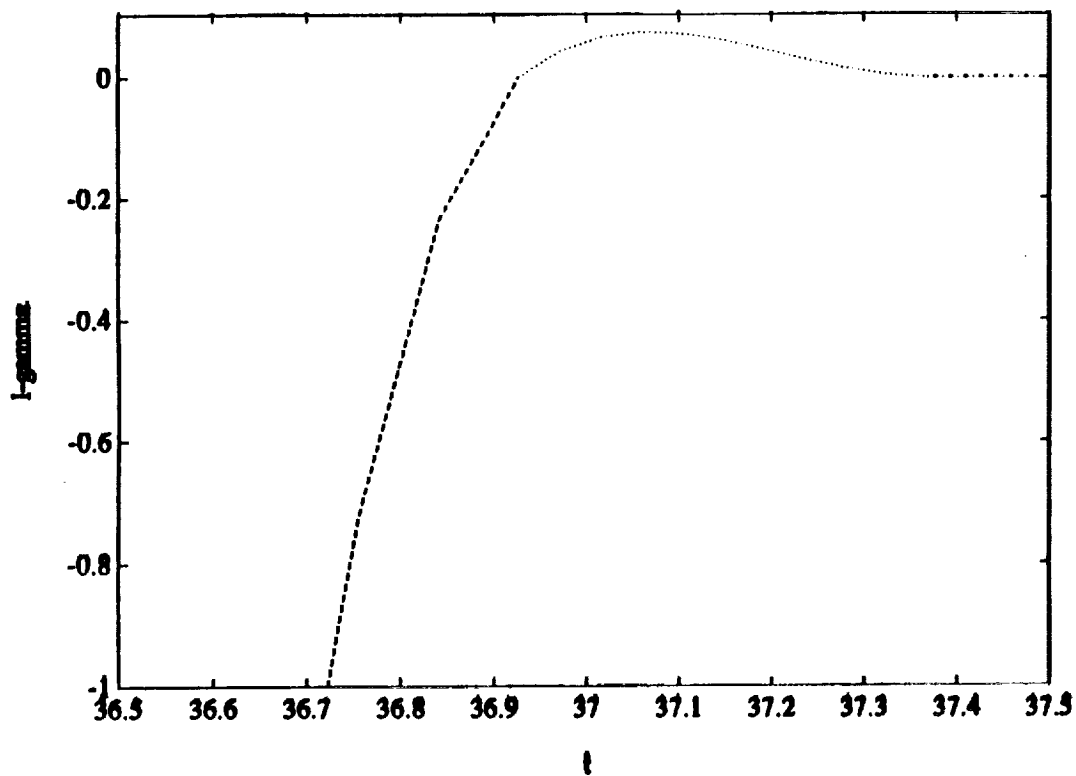
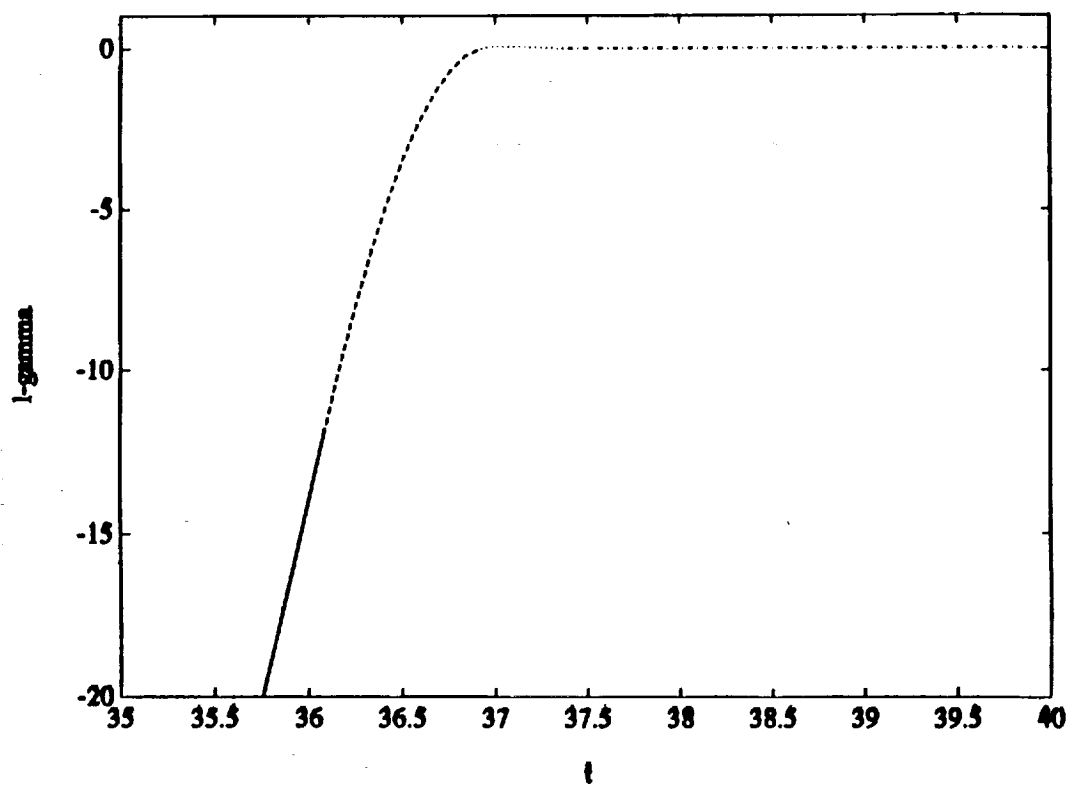












## **SUMMARY**

- All possible control logics are analyzed
- Optimal switching structures are identified.  
Solutions involve singular control along state constrained arcs
- A flexible multipoint shooting code was developed and applied successfully
- TODI was used to perform sanity check and to guess the optimal switching structure

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### **3-D Air-to-Air Missile Trajectory Shaping Study**

by

**Renjith Kumar, Hans Seywald**  
Analytical Mechanics Associates Inc., Hampton, Virginia

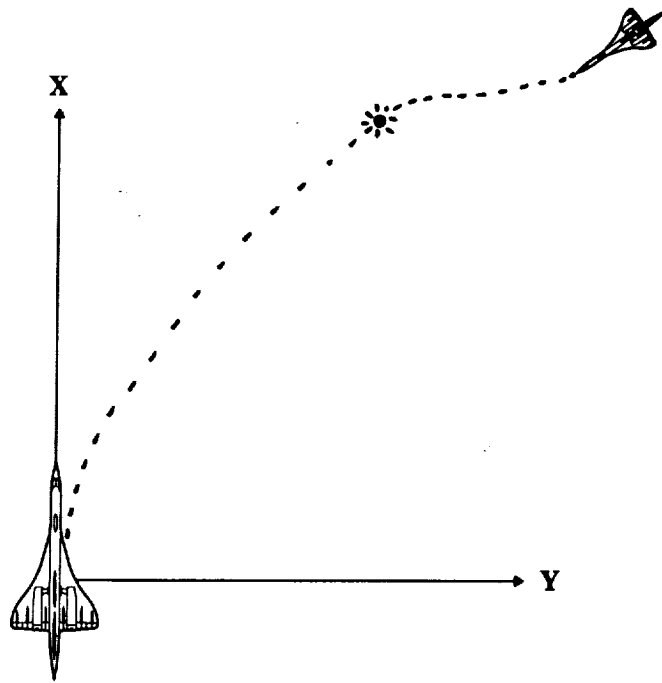
and

**Eugene Cliff, Late Henry Kelley**  
Department of Aerospace Engineering, VPI&SU, Blacksburg, Virginia

### **HISTORICAL BACKGROUND**

- Sir Francis Drake and “Manoeuvre Board”
- World War II
  - Pure-Pursuit
  - Deviated Pursuit
  - Command to Line-of-sight
  - Collision course
  - Proportional Navigation
- Singular Perturbation (Reduced-Order Modeling)

## THE GUIDANCE PROBLEM



## PROBLEM FORMULATION

### Cost

$$\min -x(t_f)$$

### Differential Constraints

$$\dot{x} = V \cos \gamma \cos \chi$$

$$\dot{y} = V \cos \gamma \sin \chi$$

$$\dot{h} = V \sin \gamma$$

$$\dot{E} = \frac{V}{W(t)} (T(t) - D(h, M, n))$$

$$\dot{\gamma} = \frac{g}{V} (n_v - \cos \gamma)$$

$$\dot{\chi} = \frac{g}{V} \frac{n_h}{\cos \gamma}$$

### Initial and Final Conditions

$$x(0) = 0 \quad x(t_f) \text{ to be optimized}$$

$$y(0) = 0 \quad y(t_f) = y_f$$

$$h(0) = h_0 \quad h(t_f) = h_0$$

$$E(0) = E_0 \quad E(t_f) \geq E_f$$

$$\gamma(0) = \gamma_0 \quad \gamma(t_f) \text{ free}$$

$$\chi(0) = \chi_0 \quad \chi(t_f) \text{ free}$$

### Controls

$$n_v, n_h$$

### Control Constraints

$$\sqrt{n_v^2 + n_h^2} \leq 30$$

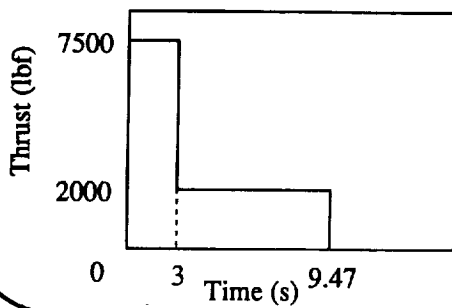
$$\sqrt{n_v^2 + n_h^2} \leq \frac{qS}{W} C_{Lmax}(M)$$

## DRAG, THRUST & WEIGHT MODEL

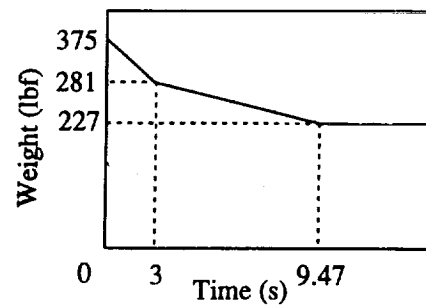
### Drag Model

$$D = qS [C_{D_0}(M, h) + C_{Di}(M) \left(\frac{W}{qS}\right)^{1.8} n^{1.8}] \quad \text{where} \quad n = \sqrt{n_v^2 + n_h^2}$$

### Thrust Model



### Weight Model



## INDIRECT METHOD

### Optimal control problem

$$\min \Phi(x(t_f), t_f)$$

$$\dot{x} = f(x, u, t)$$

$$x(t_0) = x_0$$

$$\psi(x(t_f), t_f) = 0$$

if solution does exist then it satisfies

### Boundary value problem

$$\dot{x} = f(x, u, t)$$

$$\dot{\lambda} = -\frac{\partial H}{\partial x} \quad \text{where} \quad H \triangleq \lambda^T f$$

$$\min_{u \in \Omega} H(x, \lambda, u, t)$$

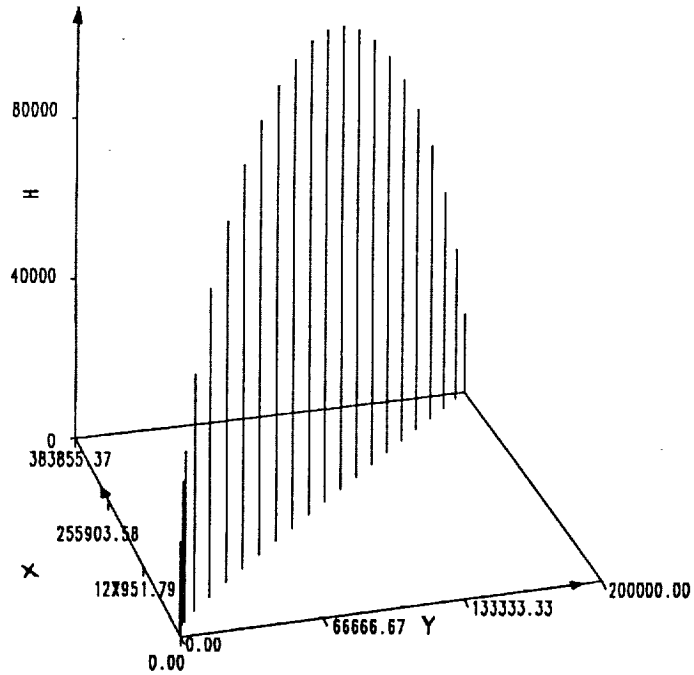
$$x(t_0) = x_0$$

$$\psi(x(t_f), t_f) = 0$$

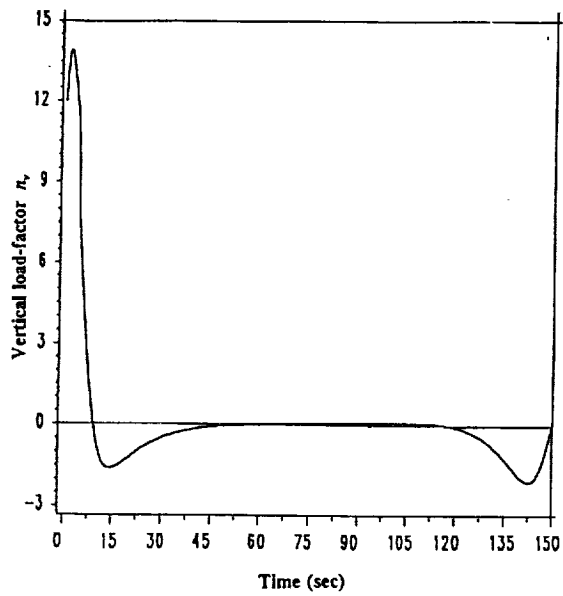
$$\lambda(t_f) = \frac{\partial \Phi}{\partial x(t_f)} + \nu^T \frac{\partial \Psi}{\partial x(t_f)}$$

$$H(t_f) = \frac{\partial \Phi}{\partial t_f} + \nu^T \frac{\partial \Psi}{\partial t_f}$$

### OPTIMAL TRAJECTORY IN 3-D

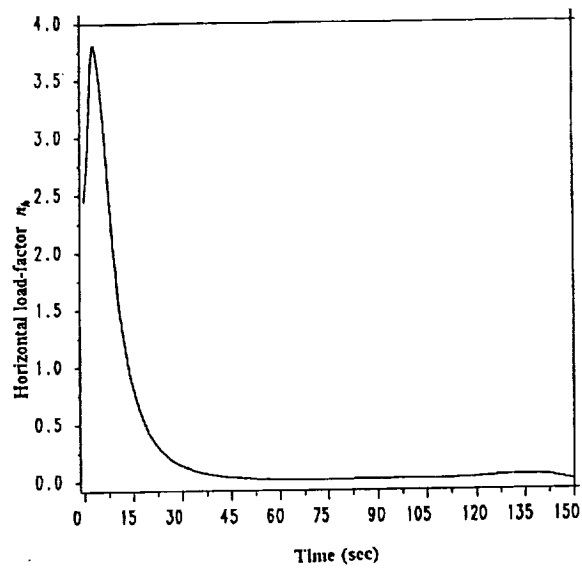


### VERTICAL LOAD-FACTOR TIME HISTORY

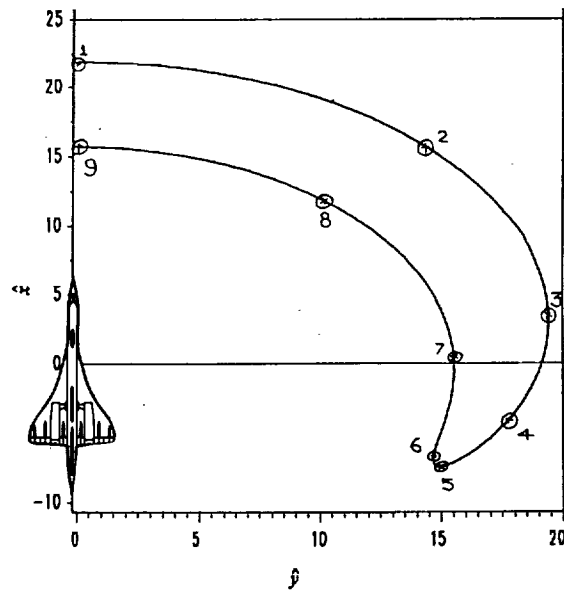




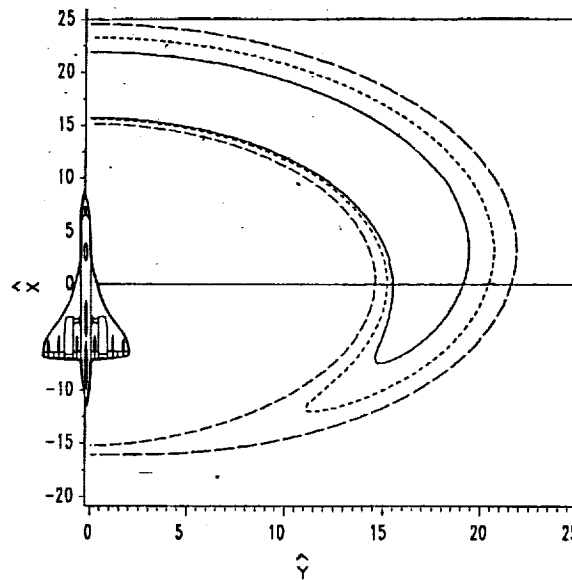
# **HORIZONTAL LOAD-FACTOR TIME HISTORY**



# **ATTAINABILITY SET FOR FINAL TIME 150s**



### ATTAINABILITY SET FOR FINAL TIME 150s, 160s, 170s



### ACCESSORY MINIMUM PROBLEM

$$\min \frac{1}{2} x(t_f)^T S_f x(t_f) + \frac{1}{2} \int_{t_0}^{t_f} [x^T, u^T] \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} x \\ u \end{bmatrix} dt$$

$$\dot{x} = Fx + Gu$$

$$x(t_0) = x_0 \quad t_0 \text{ fixed} \quad A^T = A \quad A_{22} > 0$$

$$Bx(t_f) - b = 0 \quad t_f \text{ fixed} \quad S_f^T = S_f \quad S_f \geq 0$$

### ACCESSORY MINIMUM PROBLEM (contd.)

$$u = A_{22}^{-1} [(-A_{21} - G^T(S - RQ^{-1}R^T))x - G^TRQ^{-1}b]$$

$$\dot{S} = D_{21} + D_{22}S - SD_{11} - SD_{12}S \quad ; \quad S(t_f) = S_f$$

$$\dot{R} = D_{22}R - SD_{12}R \quad ; \quad R(t_f) = B^T$$

$$\dot{Q} = -R^TD_{12}R \quad ; \quad Q(t_f) = 0$$



!! new !!

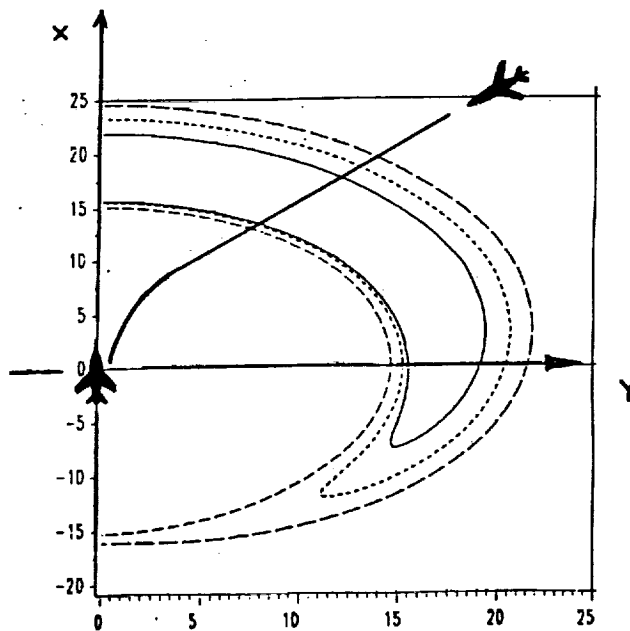
$$H \triangleq S - RQ^{-1}R^T$$

$$\dot{H} = D_{21} + D_{22}H - HD_{11} - HD_{12}H$$

### THREE PHASE GUIDANCE

- BOOST PHASE GUIDANCE
- MIDCOURSE GUIDANCE
- TERMINAL GUIDANCE

### IDENTIFY REFERENCE SOLUTION



### MIDCOURSE GUIDANCE (NEIGHBORING SOLUTION)

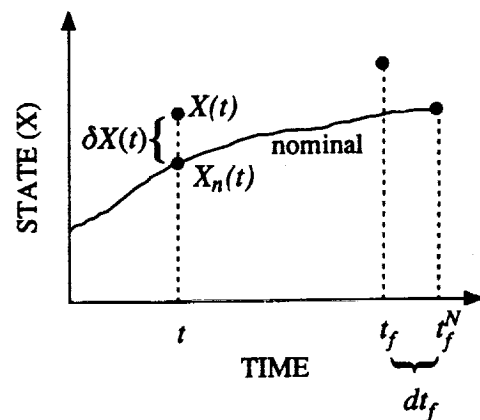
#### Closed-loop Control

$$u_{CL}(t) = u_{ref}(t) + \delta u(t)$$

$$\delta u(t) = G_1(t) \delta X(t) + G_2(t) d\psi(t)$$

#### Change in final time (cost)

$$dt_f = K_1(t) \delta X(t) + K_2(t) d\psi(t)$$



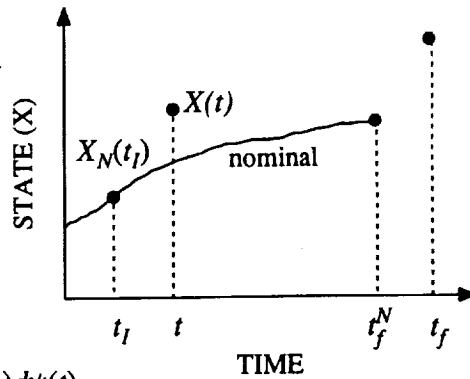
### MIDCOURSE GUIDANCE (TRANSVERSAL COMPARISON)

$$t_f' = t_f - t = t_f^N - t_I$$

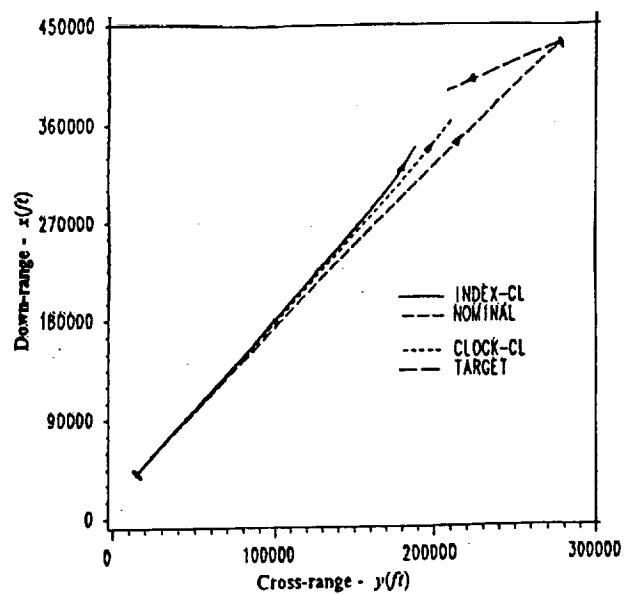
$$t - t_I = \frac{K_1(t_I)[X(t) - X_N(t_I)] + K_2(t_I)[d\psi(t)]}{[1 + K_1(t_I)\dot{X}^N(t_I)]}$$

#### Closed-loop Control

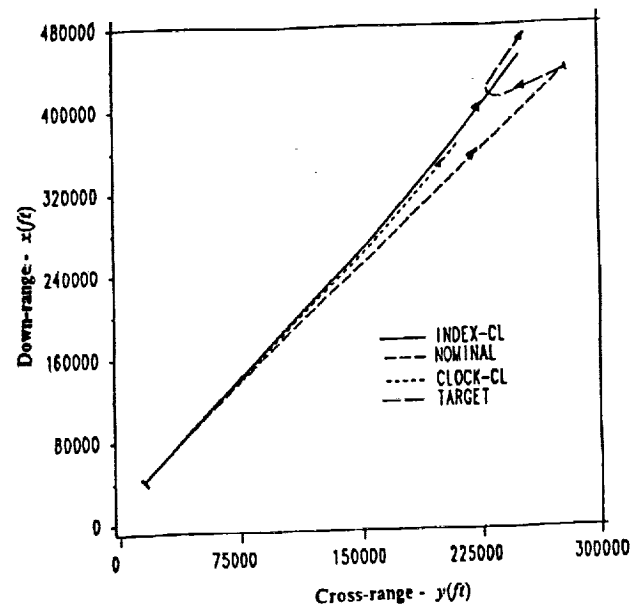
$$u(t) = u^N(t_I) + G_1(t_I)[X(t) - X^N(t_I)] \\ + [G_1(t_I)\dot{X}^N(t_I) + \dot{u}^N(t_I)][t - t_I] + G_2(t_I)d\psi(t)$$



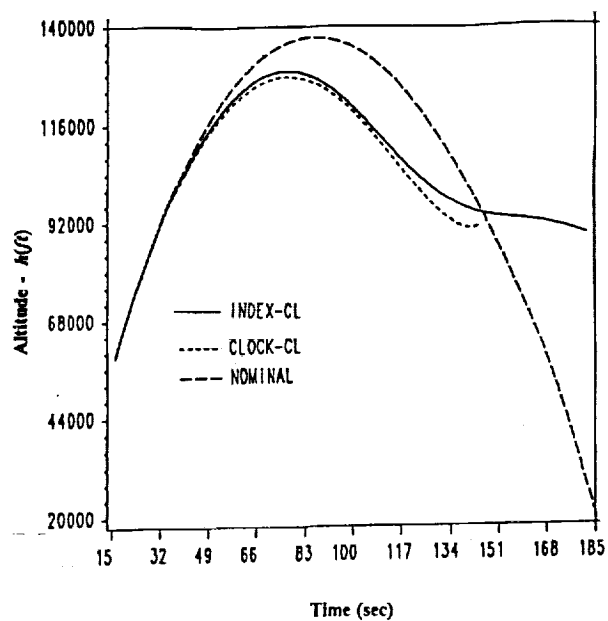
### MIDCOURSE GUIDANCE (HORIZONTAL PROJECTION) AGGRESSIVE TARGET



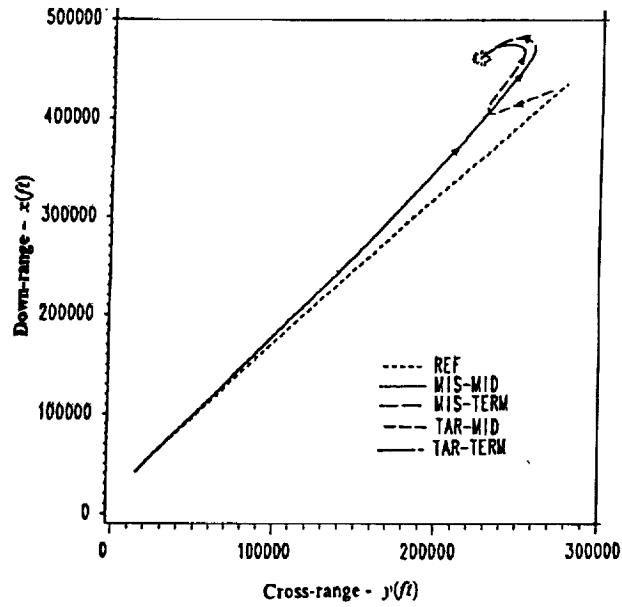
# **MIDCOURSE GUIDANCE (HORIZONTAL PROJECTION)** **RUN-AWAY TARGET**



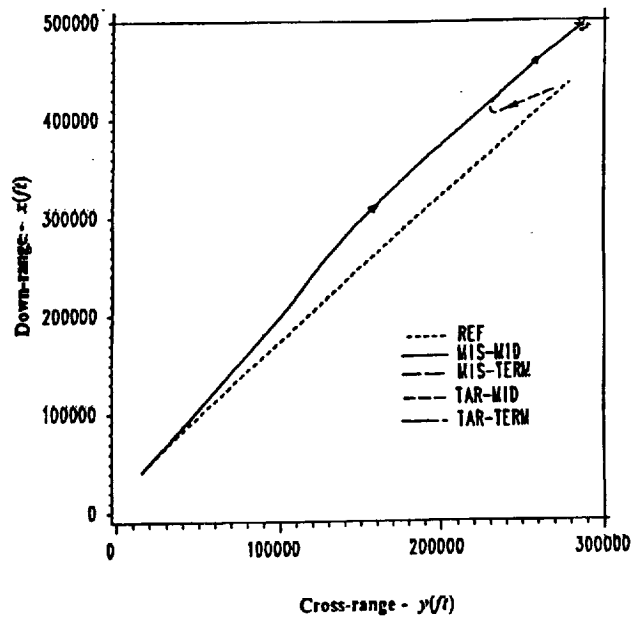
# **MIDCOURSE GUIDANCE (ALTITUDE)** **RUN-AWAY TARGET**



**NEAR-OPTIMAL GUIDANCE (HORIZONTAL PROJECTION)**  
**SHINAR'S TARGET**



**HALE-PN GUIDANCE (HORIZONTAL PROJECTION)**  
**SHINAR'S TARGET**



### **SALIENT CONTRIBUTIONS**

- Identified attainable sets via intricate homotopy procedures.
- Checked sufficiency conditions for weak local optimality.
  - Derived a new matrix differential equation for conjugate point testing.
- Developed an efficient method of optimal gain evaluation.
- Developed a composite midcourse guidance strategy (half-pn) which saves on-board storage.



# **Constrained Minimization of Smooth Functions Using A Genetic Algorithm**

**Lynda J. Foernsler, SCB  
Dr. Daniel D. Moerder, SCB  
Dr. Bandu N. Pamadi, Vigyan**

**LaRC Workshop  
March 18-19**

## **Purpose**

- **Discuss the use of a simple genetic algorithm for constrained minimization of differentiable functions with differentiable constraints.**
- **Assess the performance of this approach**

# Outline

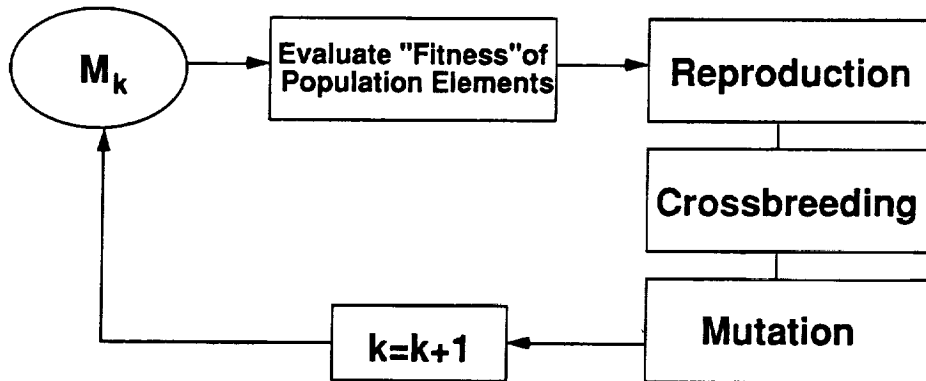
- Genetic Algorithms (GA)
- Problem Formulation for GA
- Numerical Experiment
- Comparison to Penalty Function Approach
- Conclusions
- Future Work

## Genetic Algorithms

- Nonderivative, nondescent, random search procedures for unconstrained functional minimization
- Algorithmic structure is based on notions from biology with "survival of the fittest" search heuristic
- Operations performed on successive generations of a population represented by binary coded strings (DNA-analog)

## GA Operations

**M = population**



- Initial population,  $M_0$ , is randomly generated

## Constrained Function Minimization

$$x^* = \min_{x \in \mathfrak{R}^n} c(x)$$

subject to

$$f_i(x^*) = 0 \quad i \in E$$

$$f_j(x^*) \geq 0 \quad j \in I$$

## Kuhn-Tucker (KT) Conditions

$$\begin{aligned}\frac{\partial \mathcal{L}(x, \lambda)}{\partial x} \bigg|_{x^*, \lambda^*} &= 0 \\ \lambda_j^* &\geq 0 \quad j \in I \\ \lambda_k^* f_k(x^*) &= 0 \quad k \in E \cup I \\ f_k &= 0 \quad k \in E \\ f_k &\geq 0 \quad k \in I\end{aligned}$$

where

$$\mathcal{L}(x, \lambda) = c(x) - \sum_{k \in E \cup I} \lambda_k^* f_k(x^*)$$

## Problem Formulation For GA

- Convert the solution of the necessary conditions for a constrained minimum into an unconstrained function minimization
- Solve the resulting unconstrained minimization problem

$$x^* = \arg \min_{x \in X} g(x)$$

where  $X$  is the user-specified bounded volume over which the GA takes place.

## Unconstrained Minimization Problem Formulation

$$g(x, \lambda^*) = \sum_{i=1}^n |\mathcal{L}_{x_i}(x, \lambda^*)| + \sum_{j \in E} |f_j(x)| + \sum_{k \in I} |\min\{0, f_k(x)\}|$$

- estimate  $\lambda^*$  by setting  $\mathcal{L}_x(x, \lambda) = 0$

$$\nu(x^*) = (f_x^T(x^*))^+ c_x(x^*)$$

$$\nu_i(x) = \begin{cases} \nu_i(x) & f_i = 0 & i \in E \\ |\nu_i(x)| & f_i < 0 & i \in I \\ 0 & f_i > 0 & i \in I \end{cases}$$

- KT conditions are satisfied by solving the nonsmooth equation

$$g(x, \nu(x)) = 0$$

## Genetic Algorithm Function Minimization

$$x^* = \arg \min_{x \in \mathcal{X}} g(x, \nu(x))$$

where  $\mathcal{X}$  is the user-specified bounded volume over which the genetic search takes place:

$$\mathcal{X} = \{x : (x_i)_{min} \leq x_i \leq (x_i)_{max}; i = 1, \dots, n\}$$

## GA Function Minimization

$$x^* = \arg \min_{x \in \mathcal{X}} g(x, \nu(x))$$

where  $\mathcal{X}$  is the user-specified bounded volume over which the genetic search takes place:

$$\mathcal{X} = \{x : (x_i)_{\min} \leq x_i \leq (x_i)_{\max}; i = 1, \dots, n\}$$

## Numerical Experiment (1)

- Mission: Determine control settings for an energy-state approximation of minimum-fuel ascent to orbit for the Langley Accelerator
- Control variables:

$$\bar{x} = \begin{cases} \alpha, & \text{angle of attack (deg)} \\ h, & \text{altitude (ft)} \\ \delta_E, & \text{elevon deflection (deg)} \\ \delta_T, & \text{thrust vector angle (deg)} \\ \eta, & \text{fuel equivalence ratio} \end{cases}$$

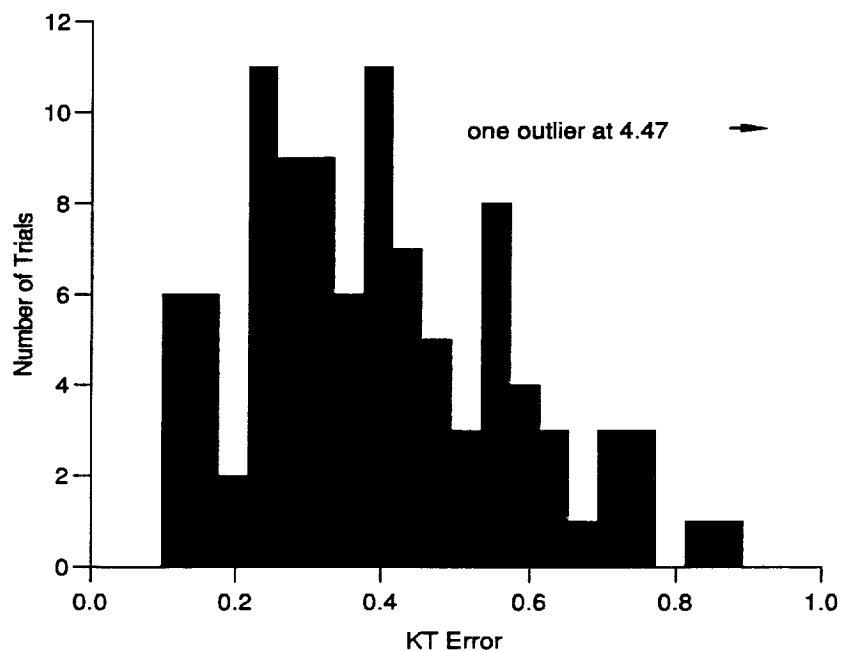
- Cost:

$$c(x) = -\frac{dE}{dm}$$

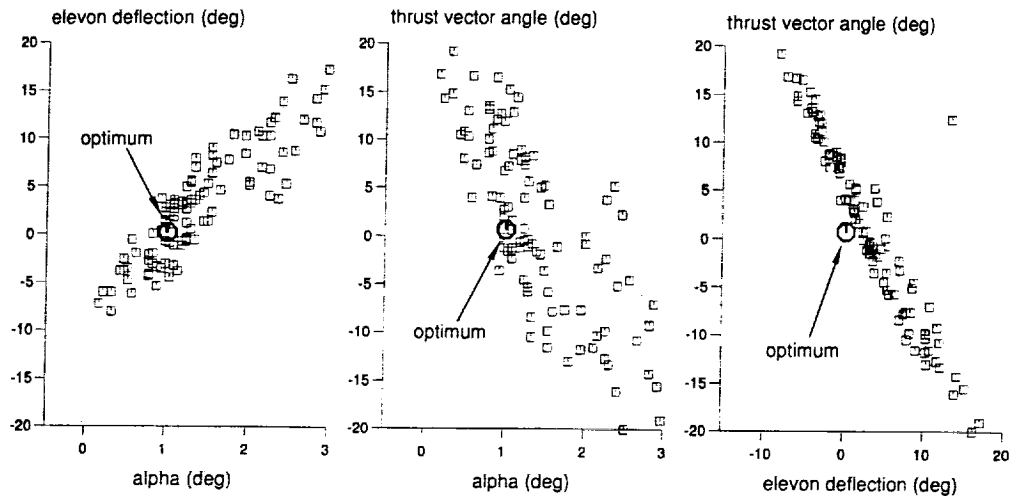
subject to

- vertical acceleration balance equality constraint
- pitch moment balance equality constraint
- dynamic pressure inequality constraint
- Monte Carlo Experiment
  - 100 GA runs
  - 600 generations/run
- Used final generation  $\bar{x}$  values from GA runs as initial guesses for Newton-Raphson (NR) method

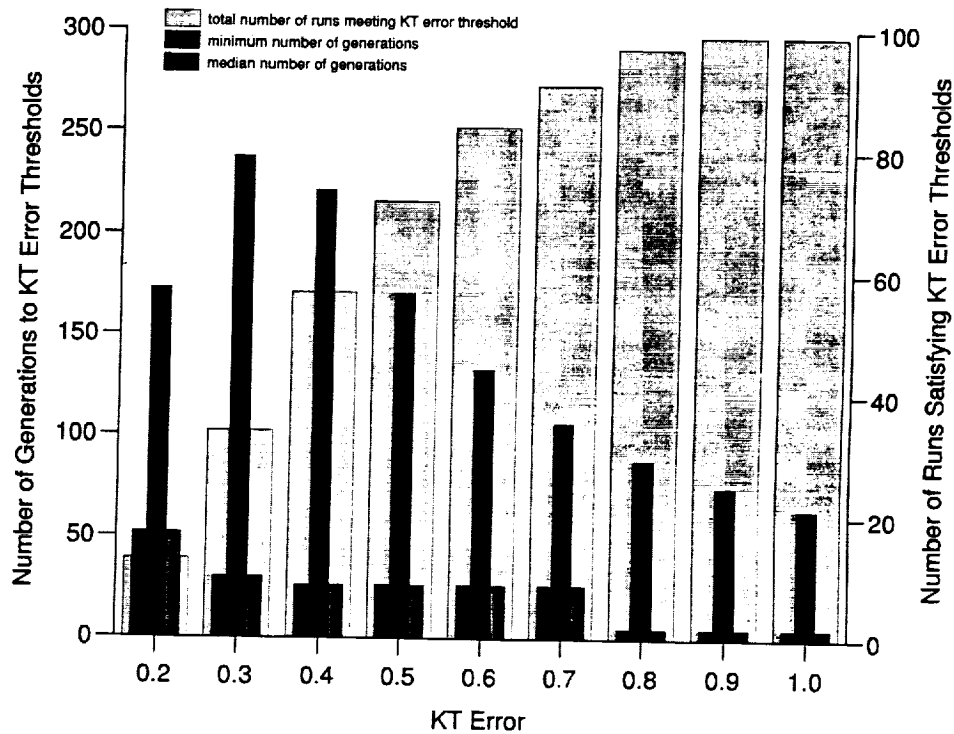
## Distribution of KT Error for Aerospace Plane Model



# Distribution of Control Settings for KT Approach



## KT Error Thresholds





## KT Formulation Results

- 82 of the NR runs converged
- 99/100 runs converged within a KT error threshold of .9
- Fewer number of generations/run would have sufficed

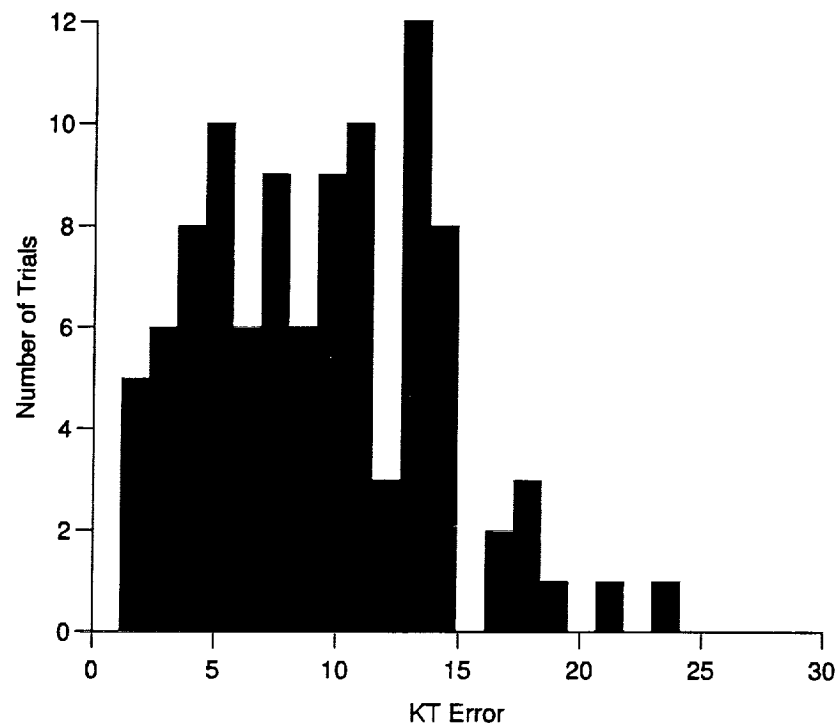
## Comparison To Penalty Approach

- Penalty function form:

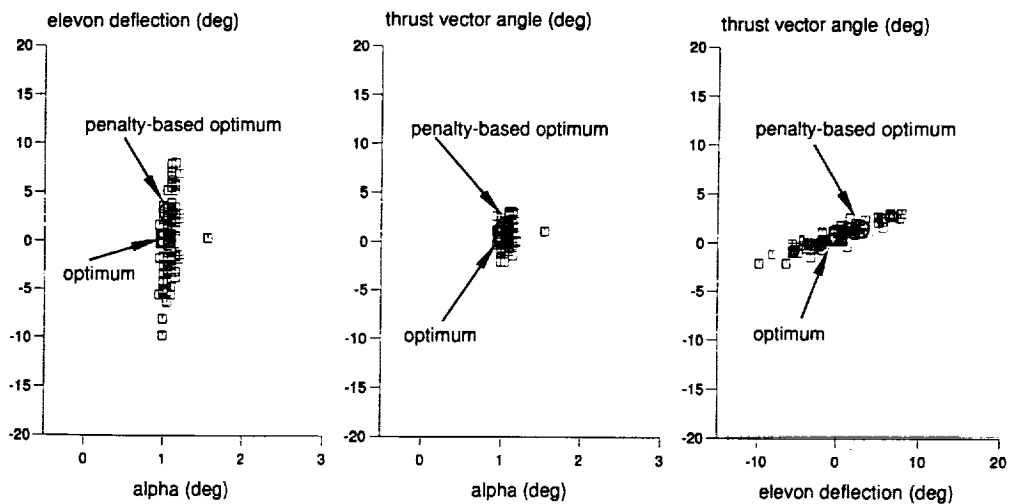
$$x_{pen}^* = \arg \min_{x \in \mathcal{X}} \left\{ c(x) + \sum_{k \in \mathcal{E} \cup \mathcal{I}} p(x, f_k(x)) \right\}$$

- Monte Carlo Experiments
  - 100 GA runs
  - 600 generations/run
  - various penalty-weighting combinations
- Initial Guesses for NR method 5 2 9

## Best Penalty Function Histogram



## Best Penalty Function Scatter Plots



## **Penalty Function Results**

- **74 of the NR runs converged for the best case**
- **fine tuning of penalty-weighting combinations is problem specific**

## **Conclusions (1)**

- **Discussed search characteristics and algorithmic operations of a simple genetic algorithm.**
- **Discussed method of adapting the KT conditions for a constrained minimization problem to formulate an unconstrained minimization function to be used by a genetic algorithm.**
- **Demonstrated KT method formulation numerically on an aerospace plane model of the Langley Accelerator**

## **Conclusions (2)**

- **For this study, KT approach provides reliable initial guesses for Newton-Raphson method**
- **Unlike the penalty approach, the KT approach**
  - **minimizes a function whose optimum value is known a priori**
  - **provides a measure of the constrained stationarity of the solution**

## **Future Work**

- **Exploit stopping criterion of KT approach**
- **Extend GA algorithm to include non-smooth cost function and non-smooth constraints**